The Barra U.S. Fixed Income Risk Model

Asked to estimate the risk of a portfolio, a manager of fixed income securities might respond by giving the duration of the portfolio, perhaps relative to the duration of a standard benchmark. To estimate both spread risk and exposure to interest rate changes, the manager might also report the durations of various portfolio components (such as the Treasury, agency, corporate, and mortgage components) compared to the corresponding subindices of the benchmark.

Duration is a well-understood, single number that identifies the portfolio's exposure to the level of interest rates, which is the largest source of market risk. Using duration to measure risk, however, leaves several unanswered questions, including the following:

- How do the various sectors interact (for example, how are mortgage spreads affected by a rise in interest rates)?
- How does the risk impact of a one-year-duration overweighting of agency bonds compare to the risk impact of a one-year-duration overweighting of BBB corporates?
- If finer measures of portfolio structure are used (such as sector-by-rating bins), how will the resulting long list of relative durations be used?
- What is the benefit of diversification? For instance, how much is credit risk reduced by using 50 bonds, compared to using just 5, to replicate the corporate index?

This paper describes Barra's approach to answering these kinds of questions in the context of the U.S. fixed income market. The section entitled "Model Structure and Estimation" provides a detailed description of the components, estimation methodology, and quantitative results of the risk model. The section entitled "Model Tests" on page 21 describes test results for the model components. The section entitled "Summary" on page 28 summarizes the results and discusses the future of the model.

Model Structure and Estimation

Barra's U.S. Fixed Income risk model[1] (USFI) includes:

- Marketwide interest rate and spread risk factors for taxable and tax-exempt fixed income securities (common-factor risk)
- Heuristic models of specific risk for U.S. government bonds (Treasuries and agency bonds) and mortgages (asset-specific risk)
- A credit migration model of specific risk for corporate, supranational, non-U.S. sovereign, and municipal bonds (issuer credit risk)

In the parametric framework, the common-factor, asset-specific, and issuer credit risk models are combined to obtain a forecast of portfolio return variance:

$$\sigma^2 = x^T F x + \sum_{\substack{Treasury,\,Agency,\\ MBS,\,ABS}} w_i^2 \sigma^2_{\substack{type(i)\\ residual}} + \sum_{other\ issuers} w_i^2 \sigma^2_{\substack{issuer\ i\\ migration}}$$

where x is the vector of portfolio factor exposures, $\boldsymbol{F}$ is the covariance matrix, w_i is the portfolio weight for each asset or issuer, and σ_i^2 is the corresponding return variance due to asset-specific factors or issuer credit migration.[2]

Common factors and asset exposures

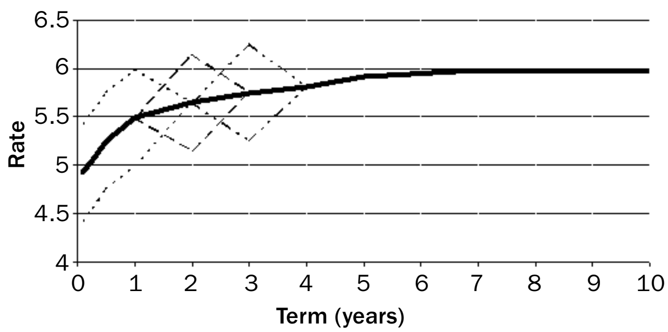
The common-factor model accounts for non-diversifiable, marketwide risk factors. The primary marketwide determinants of fixed-income portfolio risk are interest rates and yield spreads,[3] and these are the factors considered in the U.S. model. Every fixed income security in the model is exposed to one or (usually) more of these factors.

At the heart of the common-factor risk model is a covariance matrix that permits the calculation of forecasted return volatility given the portfolio's exposures to the common factors. The exposures are the analogs of ordinary effective duration for the different factors. They include key rate durations for the spot rate factors and spread duration for the sector spread factors.

Interest rates

All fixed income securities except cash are exposed to taxable and municipal market interest rates at specified maturities. Whether taxable or tax-exempt, the exposures are the so-called "key rate" durations for each vertex. Key rate durations are calculated by applying symmetric, up and down "shocks" to the initial spot rate curve, revaluing the asset with the shifted curves, and approximating the partial derivative of the asset price with respect to the spot rate from the results. Figure 1 illustrates shocked term structure shapes for the first three key rates.

Figure 1. "Shocked" spot rate curves (dashed lines) corresponding to the first three key rate durations calculated for interest rate risk exposures



The sum of the key rate durations is equal to the effective duration, since a simultaneous shock to all of the spot rates is equivalent to a parallel shift of the spot rate curve. (For this to work, the first and last key rate shifts are not tents, but are flat before the first vertex and flat after the last vertex.) Thus, key rate durations are also known as "partial" durations.

Taxable debt

Taxable securities have exposure to interest rate risk factors estimated with Treasury price data at eight spot rate maturities of 1, 2, 3, 5, 7, 10, 20, and 30 years.

Tax-exempt municipal debt

Tax-exempt (municipal) bonds have interest rate exposure to the curve estimated from AAA Government Obligation (GO) par yields using eight spot rates at maturities of 1, 2, 3, 5, 7, 10, 20, and 30 years (the same as the taxable rates).

Spreads

All fixed income securities except Treasuries and AAA municipals are exposed to a spread factor. There are nominally 61 taxable spread factors (1 agency spread, 55 corporate spreads, and 5 MBS spreads)—though not all of them can be estimated from market data—and 3 tax-exempt (municipal) spread factors.

The magnitude of the exposure to the spread factor is the spread duration. Spread duration is calculated analogously to effective duration, except that the option-adjusted spread (OAS), instead of the underlying interest rate curve, is subject to the shifts.

In some cases, such as fixed rate bonds, this makes no difference to the calculated value. In other cases, including mortgages and floating rate bonds, the spread duration and effective duration are different.

For floaters, the spread duration is comparable to the effective duration of a fixed rate bond of similar current coupon and maturity. The reason for the difference is that changing the OAS affects only the discount rate applied to future cash flows, while shifting the underlying spot rates also affects the projected cash flows.

Note: Spread duration is never negative, while effective duration can be negative in exceptional cases (such as IOs and levered floaters).

Agency bonds

There is one spread factor to which every agency bond is exposed.

Corporates, non-U.S. sovereigns, and supranationals

The spread duration of a taxable corporate, non-U.S. sovereign, or supranational bond is mapped to a spread factor by its assigned sector and rating.[4] The model includes up to 54 sector-by-rating spread factors (that is, the 9 sectors listed below times 6 rating categories from AAA to B). In addition, there is a single CCC rating spread factor across all sectors (there are not enough CCC bonds to adequately estimate spread changes for different sectors).

The sectors are:

- Canadian—Government, Provincial, Corporate
- Financial—Bank, Insurance, Independent, Subsidiaries
- Energy
- Manufacturing, retail, consumer products, diversified industrial
- Transport

- Telecommunications
- Utility—Gas and Electric
- Yankee—Supranational
- Yankee—Corporate and Sovereign (except Canada)

Mortgage-backed securities

The spread duration for a mortgage-backed security (MBS) passthrough is mapped by agency and program type to one of the five distinct mortgage spread factors below:

- GNMA 30 year
- Conventional 30 year
- GNMA 15 year
- Conventional 15 year
- Conventional balloons

Tax-exempt municipal spreads

The spread duration of a tax-exempt (municipal) bond rated AA or below is mapped to one of the three muni rating spread factors. The three muni spread factors are spreads of AA, A, and BBB GO yields over the AAA muni spot curve.

Swap spread

The spread duration of any asset exposed to credit risk but not mapped to one of the other spread factors is mapped to the swap spread factor. Assets exposed to this factor include asset-backed securities (ABSs) and collateralized mortgage obligations (CMOs).

Estimation of common-factor returns

Two basic steps are required to estimate factor returns.

1 Calculate the Treasury spot rate curve for each month of the sample period.

2 Calculate spreads for everything else (agency, corporate, non-U.S. sovereign, and supranational bonds, and generic MBS passthroughs).

Treasury curve

The Treasury curve is estimated at the eight model maturities by solving for the spot rates that minimize the mean-squared pricing error of all regular Treasury coupon notes and bonds (that is, excluding STRIPs, TIPs, and the like).

The Treasury spot rate factor returns are then:

$$r_i^{spot}(t) = s_i(t+1) - s_i(t)$$

where $s_i(t)$ is the time *t* spot rate at maturity vertex *i*.[5]

Note: The valuation algorithm accounts for option value if the bond is callable.

Option-adjusted spread

The purpose of the estimation of all the bond spread returns is to attribute the residual returns of non-Treasury bonds and MBSs to changes in marketwide OASs. Bond and mortgage OASs are calculated using algorithms based upon the one-factor, mean-reverting gaussian model of interest rates (also known as the Hull-White model). The parameters of the model are calibrated to historical interest rate movements, which typically results in an annual standard deviation of about 85 basis points for short-term interest rates, and a slightly lower standard deviation for longer-term rates. The algorithms are described in more detail in "The New Cosmos–U.S. Valuation Algorithms," *Barra Newsletter*, Summer 1997.[6]

Corporate spread

The Merrill Lynch Domestic Master and Merrill Lynch High Yield Master indices constitute the bond estimation universe for corporates. At each month end, the bonds' OASs are derived from bond prices obtained from either Bridge Information Systems or Merrill Lynch Pricing Service. The bonds are grouped by sector and rating, and the spread change for a factor is taken to be the duration-weighted average OAS change of all remaining bonds in that sector-by-rating category.[7]

Agency passthrough spread

The agency passthrough spread factors are estimated using securities in generic pools corresponding to the TBA ("to be announced") market, rather than actual pools. The TBAs are identified as the lowest-priced generics with half-point multiple net coupon and reasonably large amount outstanding. If there is more than one lowest-priced generic of a given coupon, the one with lowest OAS is taken to be the TBA for that coupon.[8] This usually results in identifying the TBAs with the most recently issued pools for each coupon.

The mortgage spread changes are calculated in the same manner as the corporate spread changes, except that the weighting factor is the spread duration rather than effective duration. (Spread duration and effective duration are the same for fixed rate bonds under the valuation methodology used, so the bond spreads are actually also weighted by spread duration in the average.)

Bond and MBS spread factor returns are then:

$$r_i^{spread}(t) = \frac{\sum_{bonds\ k} w_k(t)\left(s_k(t+1) - s_k(t)\right)}{\sum_{bonds\ k} w_k(t)}$$

where $s_k(t)$ is the spread at time t of the k^{th} bond in sector-by-rating category i, and $w_k(t)$ is the weight applied to the bond, based on duration and callability.

Muni spread

Municipal securities are exposed to the muni spot curve (estimated from the AAA GO yield curve) and possibly also to credit spreads if the securities are rated below AAA. The muni spot curve is estimated by constructing a series of synthetic par bonds (one for each annual maturity out to 30 years) and then using the same error-minimization methodology as is used for the Treasury curve. Rather than "bootstrapping" the reported yields, this more cumbersome procedure is used, because muni yields are conventionally reported for callable bonds at maturities beyond 10 years. Were the par yields bootstrapped and longer callables priced using that spot curve, they would have significantly higher yields than reported.[9] This methodology is applied to yield curves for AAA, AA, A, and BBB GO bonds. The AA, A, and BBB spread levels are then calculated at five years, and the changes in these spreads constitute the muni spread factor series. The AAA muni spot rate factor returns then are calculated analogously to the Treasury spot rate factor returns, while the muni spread factor returns are produced in the same manner as the swap spread return series.

Swap spread

The swap spread change is calculated as the change in the spread of the five-year swap rate over the five-year Treasury yield. This factor is used as a catchall proxy for the spread risk of any "spread product" not exposed to one of the corporate, MBS, or muni factors. This includes ABSs, CMOs, and LIBOR- and swap-based derivatives. The swap spread factor return is then:

$$r^{swap}(t) = s(t+1) - s(t)$$

where $s(t)$ now denotes the five-year swap spread at time t.

Covariance matrix construction

Once factor return series for all of the risk factors are available, statistical estimation and forecasting methods can be applied to them. In particular, a forecast can be produced of the covariance matrix of future factor returns. The covariance matrix can then be used to forecast the return volatility of a portfolio given the exposures of its assets to the risk factors.

Barra historical data are used to estimate the current best-guess covariance matrix of the factors. As discussed in the following sections, this entails using the following two procedures:

- Weighting recent data more heavily than older data
- Employing an effective method for handling gaps in the factor return series

Half-life estimation

Greater importance is attached to more recent market events than to older ones by exponentially weighting the factor series in the covariance calculation. Each factor return is weighted by a multiplier λ (between 0 and 1) raised to the power of the number of elapsed months since the return. The value of λ can be related to the half-life of the weighting scheme, that is, the number of months n such that λ^n=1/2.

This approach is tested by evaluating the out-of-sample performance of covariance matrices constructed using different half-lives n. The probability of observing each month's factor returns given the prior month's covariance forecast (for a given half-life) is calculated, and the probabilities for successive months are multiplied together. This provides the correct probability for the entire series of returns, assuming that factor returns (that is, spot rate and spread changes) in different months are independent.

The probability numbers are kept intelligible by reporting the negative logarithm of the likelihood. These quantities are also normalized by subtracting the minimum value over the range of tested half-lives. An increase in this statistic by 1 over the minimum corresponds to a decrease in probability by a factor of $1/e \approx 0.37$.[10]

Figure 2 shows this analysis applied to Treasury spot rates, with an in-sample period of January 1996 through December 1999. Data from 1988 through 1995 were used to construct the covariance matrices, but they were not included in the log-likelihood calculation. The curve shows that the negative log-likelihood of the observed spot rate changes as a function of half-life. There is a minimum of approximately two years, but longer half-lives are not strongly excluded. Half-lives under approximately one year, however, are clearly rejected. (If data back to 1983 were used in the covariance matrix estimation, longer half-lives would be more strongly rejected. The best value remains around two years.)

Figure 2. Negative log-likelihood of observed Treasury spot rate monthly changes as a function of half-life in the covariance calculation. The best fit is about two years.

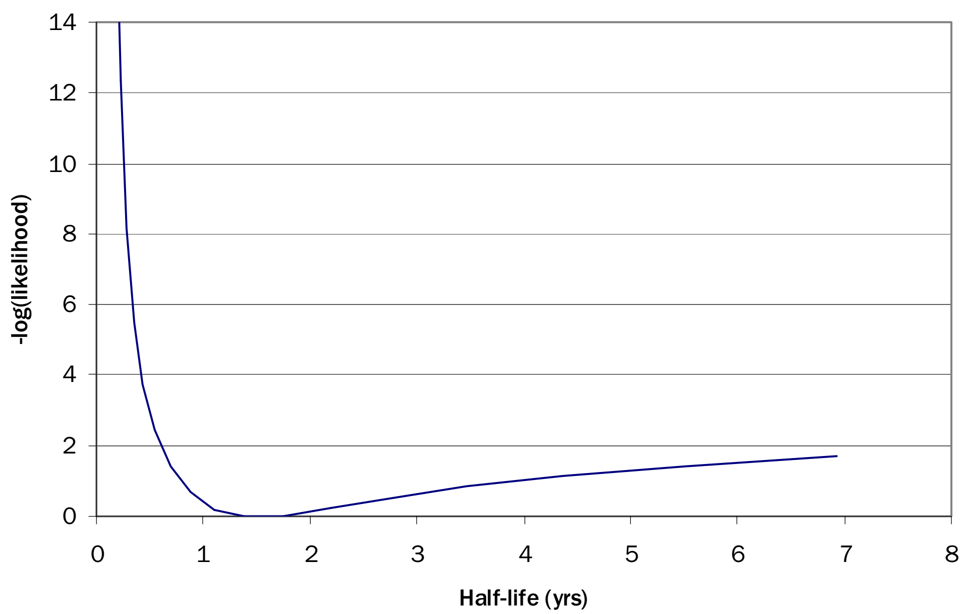
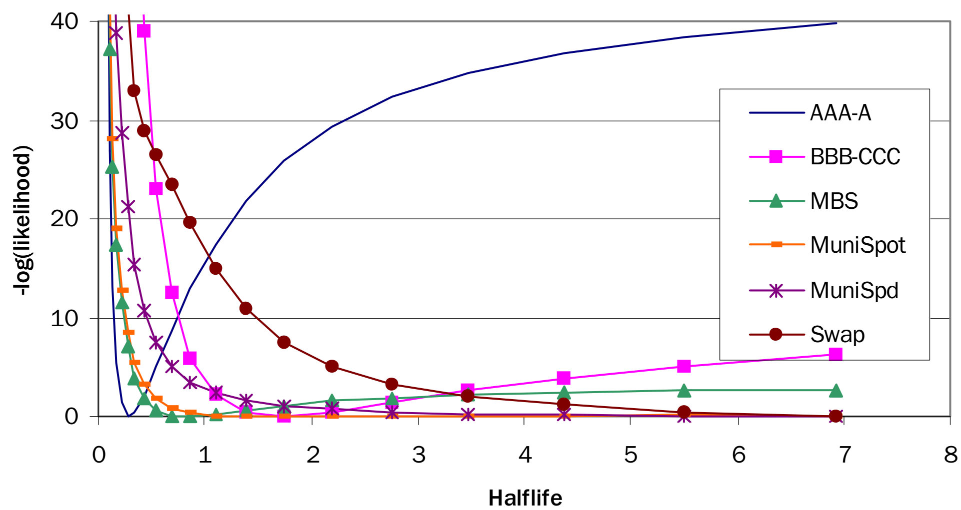


Figure 3 shows the same analysis applied to agency and corporate spreads, MBS spreads, muni spot rates, credit spreads, and swap spreads. In each case, the curve shows an average of the negative log-likelihoods for the factors in each category. The optimal forecasting half-life is long for all but the high-grade corporates, for which the best fit is a fairly short four months.

Figure 3. Negative log-likelihood of observed spread and muni factor changes as a function of half-life in the covariance calculation. The best fit is a few months for high-grade corporates, one year for MBS, and too long to measure with the sample data for the others.



Unfortunately, it is difficult to accommodate different half-lives for different factor series in a single covariance matrix. A single half-life must be selected based upon both the results of the tests and the following more qualitative considerations:

- Longer half-lives give more stable forecasts with smaller statistical uncertainty and random fluctuations.
- The risks of a portfolio with primarily investment grade securities are likely to be determined largely by the interest rate factors, for which the optimal half-life is around two years.
- The risks of a portfolio with heavy exposure to lower grade securities, for which spread risk may be dominant, are optimally forecast with the longest half-life.
- The Barra Global Fixed Income risk model is already in production using a half-life of two years (based upon similar tests across many fixed income markets).
- Finally (as described later), the two-year half-life worked quite well in the aftermath of the liquidity crisis of late 1998, which saw large short-term jumps in spreads, and where volatilities were as much as 10 times higher than previous levels. The average spread volatility has since settled down to a level about twice as high as it was prior to the 1998 events, and the model has tracked this behavior with remarkable fidelity.

Incomplete data methodology

When working with complete time series (that is, where there are neither missing values nor incomplete series), it is easy to calculate the optimal estimator of the covariance of the i^{th} and j^{th} factors; it is given by the usual pairwise formula from basic statistics:

$$C_{ij} = \sum_{t=1}^{N} \frac{\left(r_i(t) - \overline{r_i}\right)\left(r_j(t) - \overline{r_j}\right)}{N-1}$$

This formula cannot be used, however, when the factor series are of different lengths N or have missing values. Calculated in this naive fashion for such series, the resulting covariance matrices may imply that certain portfolios have negative return variance, which is mathematical nonsense. In other words, this simple formula does not give a good estimate (or even a valid one) for the covariance matrix when the factor series have gaps or are otherwise incomplete.

There are two sources of incompleteness in the factor return series. One is that for each series there is no information prior to some date. For example, the taxable spot rate and spread series begin August 1992, while the municipal data begin January 1993.

The second source of incompleteness is that many of the series contain gaps. These gaps usually arise because of the absence of primary data (for instance, when there are too few bonds to estimate a sector-by-rating category's spread change). Some of these gaps can be remedied by proxy. For example, if there are too few bonds to estimate the Utility-B spread change, the Utility-B spread change can be proxied with the average spread change for all bonds with a B rating.

A more elaborate statistical tool is required to cope with missing data that cannot be proxied. The standard technique is known as the EM algorithm. The EM algorithm iteratively converges to the maximum likelihood estimator of the underlying covariance matrix given incomplete factor return series.[11]

Unlike the naive method for incomplete data, the EM algorithm produces valid covariance matrices (that is, the matrices do not imply any negative portfolio return variances), and the algorithm produces the same covariances as the naive formula for all factor return pairs with complete data. A major benefit of using this method is that data for which there is a long history do not have to be thrown out in order to add a new, shorter series. The best covariance estimate possible is derived given all of the available data.

Common-factor model results

Figure 4 shows the month-by-month averaged volatility forecasts (annualized) for factors in different groups. For example, the curve labeled "AAA" represents the average volatility forecast for all AAA sector spreads. The jump in forecasted spread volatility in late 1998 is plainly evident, while spot rate volatilities have barely moved.

Figure 4. Volatility forecasts from the U.S. Fixed Income risk model averaged over spot rate vertices and across sectors by rating

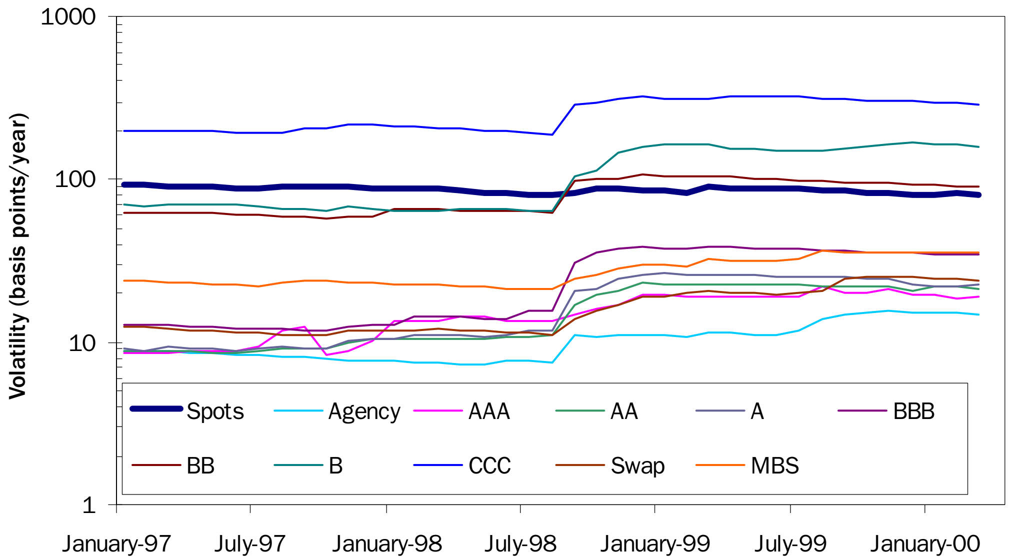


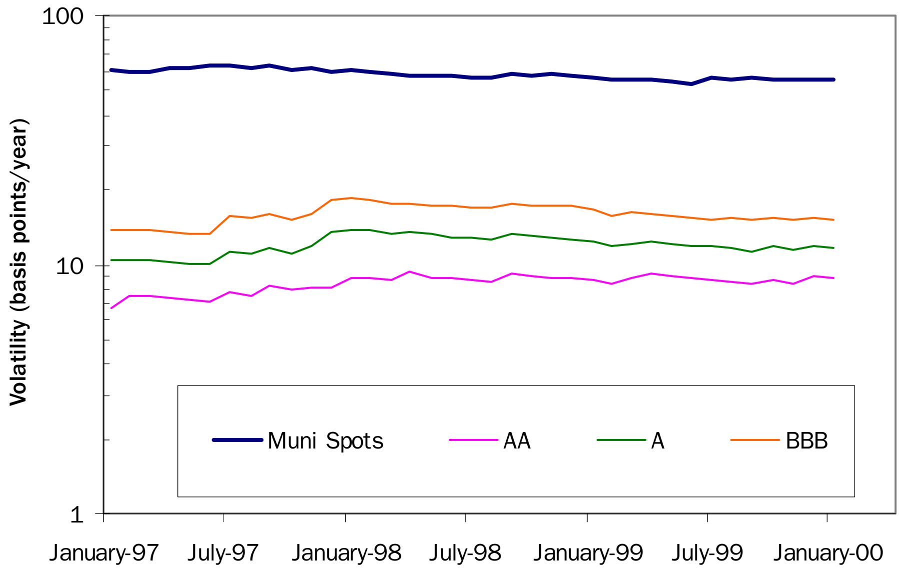
Table 1 compares average volatility forecasts from July 1998 to January 2000.

Table 1. Factor volatilities (in basis points) pre- and post-August 1998

Factor Group	Jul 98	Jan 00
Spot Rates	81	82
Agency	8	15
AAA	14	20
AA	11	22
A	12	22
BBB	15	34
BB	64	92
B	65	164
CCC	194	296
MBS	22	35
Swap	11	25

Muni bond factor volatility forecasts are shown in Figure 5. The events of 1998 and the subsequent increased volatility in the taxable credit markets appear to have had modest impact on the tax-exempt market. Muni spot rates are about 70% as volatile as Treasury spot rates (which one would expect if muni rates were typically determined relative to taxable bond rates). The correlation of muni rates with the corresponding Treasury rates is actually only about 0.6, so at least over monthly horizons the rates are not closely linked. The magnitude of volatilities, however, seems to bear out the existence of the relationship one would naively expect based on tax rates over the longer term.

Figure 5. Municipal bond factor volatility forecasts



Specific risk and credit risk models

The marketwide interest rate and spread factors discussed earlier account for the non-diversifiable portion of return risk. Each asset is additionally subject to diversifiable risk due to issuer- or asset-specific events, such as supply imbalances (squeezes on Treasury issues), defaults, corporate actions, changes in issuer credit quality, shifts in homeowner refinancing behavior, and so on. These risks are dealt with in an aggregate fashion through the specific risk and credit risk models.

The first is an empirical model of residual asset spread changes after accounting for the common factors. This type of model is used for Treasuries, agency bonds, mortgages, and structured products.

The second is a model of issuer credit risk based upon historical credit migration rates. This model is used for corporates, non-U.S. sovereigns, and supranationals (that is, all bonds other than Treasuries, agency bonds, mortgages, and structured products).

Specific risk model

For Treasuries, agency bonds, ABSs, CMOs, and MBS passthroughs, specific risk is modeled by determining empirically how much actual spread volatility is left unexplained by the covariance matrix. This additional spread volatility is included in the asset-level calculation of risk, and it is assumed to be fully diversifiable. (That is, it is assumed that the returns of different assets not explained by the market factors are independent of each other.) This is a good assumption if the marketwide factors have been well identified.

For these securities, the specific risk model has the form:

$$\sigma_{spec}^2 = a_{type}^2 D^2$$

where a is a constant estimated for each security type, and D is the security's spread duration. (In a linear approximation, the constant a is the specific risk expressed as a spread volatility.) These variances are then averaged by asset weight in the portfolio to get the overall contribution to specific risk.

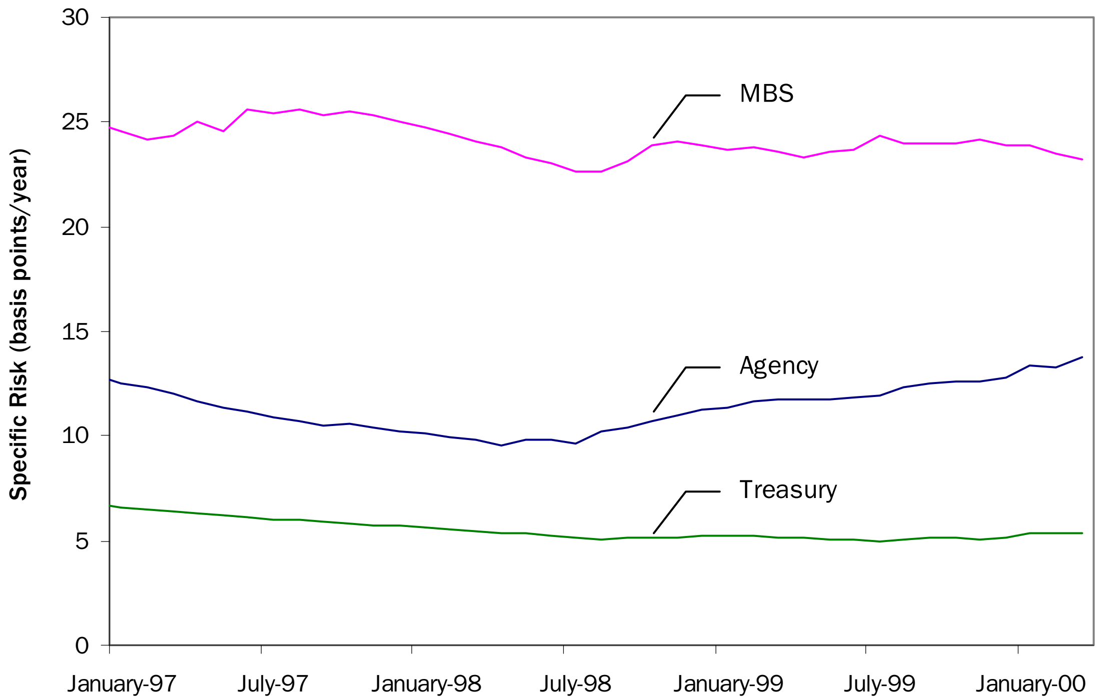
Estimation of specific risk parameters

The specific risk model for Treasuries, agency bonds, and MBS pools incorporates the same exponential weighting scheme as does the covariance matrix estimation. The standard deviation of the residual spread changes (asset spreads or OASs) of the assets is calculated relative to the common-factor spread. The common-factor spread change for Treasuries is taken to be 0, since there is no Treasury spread.

In the common-factor model, more recent data are weighted more heavily than older data, with a two-year half-life. This approach makes the model responsive to changes in market behavior without producing forecasts that are, themselves, excessively volatile.

Figure 6 shows the model's specific risk forecasts for Treasuries, agency bonds, and MBS pools since 1997, expressed as standard deviation of residual spread changes. The period prior to mid-1998 was one of falling specific return volatility, while there has been a sharp increase in the volatility since then, though more strongly in the bond sector than for MBS pools.

Figure 6. Specific risk forecasts expressed as annualized spread volatility



Credit risk model

For bonds subject to credit risk, namely those of corporate, non-U.S. sovereign, and supranational (hereafter collectively abbreviated to "corporate") issuers, a different approach is used, namely, a model of credit risk itself, rather than just a statistical model of its impact on returns. The model is very similar to J.P. Morgan's CreditMetrics.

Both Standard and Poor's and Moody's produce annual reports showing the historical rating change experience for all issuers that they follow. This information can be used to produce a transition matrix, where each entry is a probability that represents the chance that a bond with a rating of X today will have a rating of Y in a month or a year. If rating Y is different from rating X, it is reasonable to forecast that the bond's spread also will have changed significantly.[12] This spread change is uncorrelated with marketwide spread changes for the issuer's sector and initial rating, so it is an independent contribution to total risk.

If the history of rating changes is a reasonable forecast of their future likelihood, the transition matrix and the estimates of spreads-by-rating can be used to forecast the contribution of changes in credit quality to the volatility of bond spreads. It can also be used to capture the risk of rare, "long-tailed" events, such as the default of an A- or higher-rated issuer, even though data on such events are not found in Barra's bond database.[13] The transition matrix reflects the small probability of such events, and these events make a non-trivial contribution to the specific risk for investment-grade bonds. High-yield bonds are even more exposed, both to the risk of default and to large spread moves from changing credit quality, and the transition matrix method predicts proportionately large credit risk for these bonds.

Barra's U.S. Fixed Income risk model takes the raw rating transition matrix produced by S&P as an empirical sample, subject to measurement error. It is then used to find a better estimate of the "true" transition probabilities. [See "Estimation of credit risk parameters" on page 17.]

Given the matrix $\boldsymbol{M}_{fi}$ of probabilities for transition from rating i at the beginning of a month to rating f at the end, and spread levels s_i and s_f for each rating, the approximate return variance for a bond due to credit migration can be calculated as:

$$\sigma^2_{credit} = D^2 \sum_f M_{fi}(s_i - s_f)^2 + M_{di}(1-R)^2$$

where D is the spread duration, and the sum excludes transitions to default; $\boldsymbol{M}_{di}$ is the probability of a default transition; and R is the estimated recovery rate. With the addition of a small additional term to account for credit migrations within a broad rating category (such as from AA+ to AA), this is the parametric formula for the credit risk of a bond. A representative value of 50%, typical of recovery rates for senior debt, is used for R.

The credit risk formula is very responsive to changes in market expectations, due to the dependence of σ^2 upon current spread levels.[14] In circumstances where low-grade credit spreads widen out significantly relative to high-grade credit spreads (due perhaps to expectation of increased default risk), the credit risk forecast would increase immediately by a corresponding amount.

The credit migration risk is treated as perfectly correlated for all debt from each issuer, because a credit event affects all of an issuer's bonds jointly. In the parametric context, then, the main difference in the calculation method between the empirical specific risk (for Treasuries, agency bonds, ABSs, CMOs, and MBS passthroughs) and the specific risk due to credit migration (for corporates) is that the contributions of the former add independently (that is, are diversified) at the security level, while the contributions of the latter add independently at the issuer level.

Estimation of credit risk parameters

Each year (usually in January), Standard and Poor's produces a report that summarizes the events of the preceding year in the bond market from the rating agency's perspective. Included in this report is a table showing the average observed credit migration probabilities since 1981. Table 2 reproduces the table found in the January 26, 2000 issue of S&P Credit Week. The entries give the observed fraction of transitions from the rating category in the left column to the rating category in the top row, averaged over all one-year periods in the sample.

In other words, each entry in the table shows the fraction of issuers rated *X* (down the left column) at the beginning of a one-year period with rating *Y* (across the top row) at the end of that year. The row labeled AA, for example, indicates that 88.65% of the issuers in the sample rated AA at the beginning of a year were also rated AA at the end, while 3.42% of these AA issuers became non-rated.

Table 2. Rating migration rates, from Standard and Poor's Credit Week, January 26, 2000

	AAA	AA	A	BBB	BB	B	CCC	D	N.R.
AAA	89.61	6.61	0.40	0.10	0.03	0	0	0	3.24
AA	0.58	88.65	6.55	0.61	0.05	0.11	0.02	0.01	3.42
A	0.06	2.28	87.48	4.72	0.47	0.21	0.01	0.04	4.73
BBB	0.03	0.24	5.05	83.04	4.33	0.80	0.12	0.21	6.18
BB	0.03	0.10	0.43	6.43	74.68	7.13	0.99	0.91	9.30
B	0	0.11	0.28	0.49	5.36	73.81	3.48	5.16	11.33
CCC	0.14	0	0.28	1.12	1.54	9.13	53.09	20.93	13.76

A matrix derived from this one is used to forecast issuer risk as described earlier. This entails one major assumption, and it requires some "improvements" to the matrix. [See "Forecasting the one-month credit transition matrix" on page 31.] The major assumption underlying this model is that the transition probabilities do not vary over time (that is, an issuer rated AA on January 1, 1990 would have the probabilities given by the table of the various transitions during the year to January 1, 1991, as would another AA issuer on January 1, 2000.) This assumption is, perhaps, a weak link in the model; default rates, for example, are observed to go through cyclical ups and downs, rather than being constant.

Given this assumption, the observed transition probability matrix can be modeled as being the result of a year's worth of continual rating changes, denoted by $M_{fi}(1\ year)$, with subscript f representing the final rating and i the initial rating. It is then possible to show that M is related to the instantaneous transition *rate* matrix Λ by $M(T)=exp(\Lambda T)$, where the entries of the matrix Λ_{fi} give the rate of migration to rating f from rating i. The matrix exponential is defined by its power series expansion.

Conversely, given M, the transition rates Λ can be found by $\Lambda = log(M(T))/T$, where $log(M(T))$ is again defined by power series expansion, in this case around $M(T)=1$.[15] From the improved migration rate matrix Λ, the one-month transition *probability* matrix $M_{fi}(1\ month)$ can be derived using the exponential formula.

For illustrative purposes, credit risk can be more conveniently expressed in terms of spread risk by dividing out by the squared spread duration D and taking the square root to obtain a standard deviation. Figures 7 and 8 show the calculated credit risk over time for a bond with spread duration of five years, assuming a 50% recovery rate R. Sharp jumps in risk forecasts for all rating classes are apparent in mid-1998 at the time of the credit crash, due to the significant widening of spreads. The forecasts have subsequently fallen, though not for the most part back to their pre-July 1998 levels.

Figure 7. Credit risk forecasts for a five-year-duration bond, by rating (AAA to BBB), based upon the transition-matrix model. The Treasury market-specific risk forecast has been included for comparison.

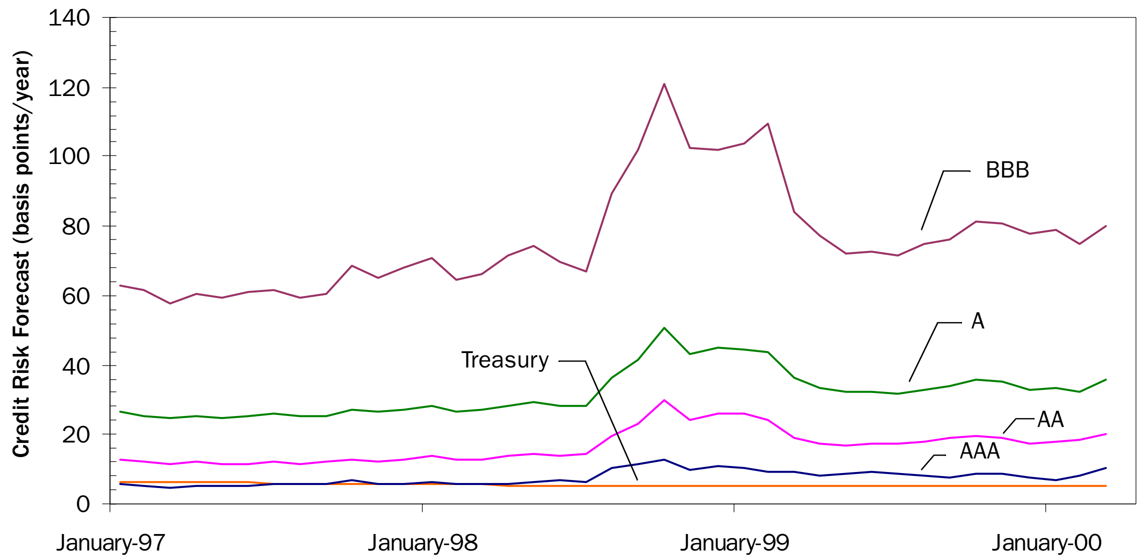


Figure 8. Credit risk forecasts for a five-year-duration bond, by rating (BB to CCC), based upon the transition-matrix model

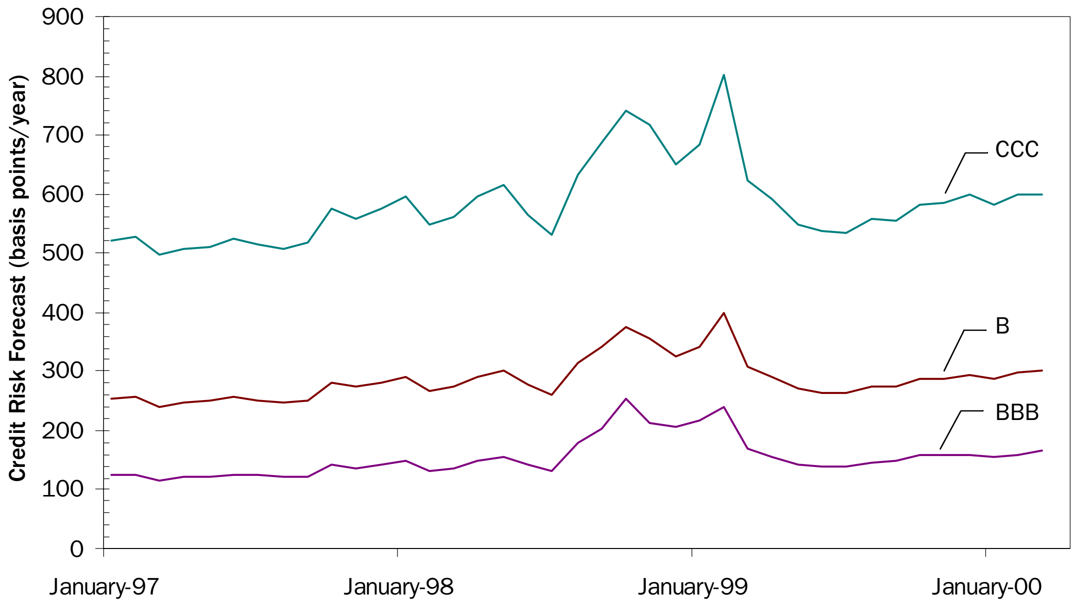


Table 3 summarizes the model forecasts for January 2000. The first column is the same as the second column of Table 1. The second column shows the specific or credit risk forecast for a single security. For the rating-based credit risk forecasts, a spread duration of five years is assumed.

Table 3. Common-factor and specific risk forecasts (as of January 31, 2000) expressed in basis points as annualized standard deviation of interest rates or spreads

	Common-Factor	Specific/Credit
Treasury/Spot	82	8
Agency	15	14
AAA	20	7
AA	22	18
A	22	33
BBB	34	79
BB	92	155
B	164	287
CCC	296	582
MBS	35	21
Swap	25	N/A

An interesting observation from this table is that the classification of issuers into investment grade (BBB and above) and speculative grade (below BBB) neatly corresponds to the split between bonds whose common-factor and credit risk are each less than their interest rate risk and those for which they are greater. That is, the common-factor and credit risks for ratings of BBB and above are all less than the 82 basis points of risk due to spot rate volatility, while those of lower-grade bonds are above this level.

Model Tests

The key test of a risk model is how well it predicts risk. To be meaningful, such a test must be run on out-of-sample data.

A "bias test" is used to assess this performance. This test looks at out-of-sample standardized returns and compares their standard deviation to the expected value of 1. If the risk forecast is accurate, then the time series formed by taking a return series and dividing each return by its predicted volatility should have a standard deviation of 1.

In practice, due to finite sample sizes, the actual sample standard deviation will itself be distributed around 1 in a way that can be calculated. If b denotes the "bias statistic," calculated as $b = \text{stdev}\left(r_i / \sigma_i\right)$ for the return series r_i with N observations (assuming that the returns are independent and that the risk forecasts σ_i are accurate), then b is distributed according to the chi-squared distribution as: $P(b) \sim \chi^2_{N-1}(Nb^2)$.

This test has been applied to the components of the risk model. The test period included the credit crash of August–November 1998, a period during which the model forecasts for spread risk were significantly too low (by factors of 2 to 10). To get meaningful results, therefore, the tests have been split into two groups: pre-crash and post-crash. The post-crash group, in particular, provides a sensitive test of the model's responsiveness to credit markets changes since 1998 (up to June 2000). The measure of statistical significance is the 95% confidence limit. From the χ^2 distribution, this means that b should be in the range of $1 \pm \sqrt{2/N}$, where N=10 or more sample events.

Tests of the common-factor model

For most of the factors, both pre-crash and post-crash, the model volatility forecasts are within the expected range.

Interest rate volatility

Interest rate volatility is well forecast over the entire sample period, including the 1998 crash [see Table 4]. During the pre-crash sample period, the one-year spot rate volatility is over-forecast by a statistically significant 60%. While the volatilities of the other spot rates are also over-forecast, the errors are not significant. On average over the full four-and-one-half-year test period, the spot rate volatility forecasts were 13% greater than the realized volatilities.

Table 4. Model bias statistics for U.S. spot rates (the 95% confidence intervals for unbiased forecasts are shown in parentheses at the top)

Factor	Jan 96–Jul 98 (1±0.26)	Dec 98–Jun 00 (1±0.32)	Jan 96–Jul 00 (1±0.19)
SPOT_1	0.61	0.84	0.82
SPOT_2	0.75	0.84	0.87
SPOT_3	0.81	0.75	0.86
SPOT_5	0.82	0.88	0.89
SPOT_7	0.86	0.79	0.93
SPOT_10	0.87	0.89	0.94
SPOT_20	0.94	0.86	0.90
SPOT_30	0.86	0.82	0.88

Factor volatility

As shown in Table 5, most of the factor volatility forecasts for the corporate, agency, and swap factors are unbiased, though 37% of pre-crash and 28% of post-crash factor volatility forecasts are outside the statistically expected range. They are not too far outside, however, with the exception of the post-crash Yankee BB spread. That the accuracy of the forecasts improved following the 1998 events is remarkable, considering that the forecasts jumped approximately by a factor of two from June to December of 1998.

Table 5. Pre-crash and post-crash bias statistics for agency, corporate, and swap spread factors. (Some factor series are missing due to data limitations, and shading indicates a bias statistic outside the 95% confidence interval shown in parentheses at the top.)

Factor	Jan 96–Jul 98 (1±0.26)	Dec 98–Jul 00 (1±0.32)
AGENCY	0.64	1.67
FINANCIAL_AAA	0.84	1.27
INDUST_AAA	1.01	1.05
SUPRANTL_AAA	0.90	1.49
TELE_AAA	1.30	0.98
YANKEE_AAA	1.01	0.76
CANADIAN_AA	1.03	1.15
ENERGY_AA	1.06	0.75
FINANCIAL_AA	1.38	0.94
INDUST_AA	1.23	1.02
SUPRANTL_AA	1.12	1.50
TELE_AA	1.48	1.15
UTILITY_AA	1.40	1.39
YANKEE_AA	0.85	0.79
CANADIAN_A	0.71	1.05
ENERGY_A	1.05	1.22
FINANCIAL_A	1.48	0.95
INDUST_A	1.10	1.30
TELE_A	1.45	1.39
TRANSPORT_A	1.00	1.01
UTILITY_A	1.19	1.52
YANKEE_A	1.27	0.82
CANADIAN_BBB	0.70	0.74

Factor	Jan 96–Jul 98 (1±0.26)	Dec 98–Jul 00 (1±0.32)
ENERGY_BBB	1.24	1.10
FINANCIAL_BBB	1.08	0.87
INDUST_BBB	1.18	1.24
TELE_BBB	1.51	1.49
TRANSPORT_BBB	0.95	1.54
UTILITY_BBB	0.55	1.25
YANKEE_BBB	1.79	0.70
CANADIAN_BB	0.77	1.95
ENERGY_BB	0.84	0.90
FINANCIAL_BB	0.65	0.81
INDUST_BB	0.98	0.97
TELE_BB	0.47	1.02
TRANSPORT_BB	0.57	0.92
UTILITY_BB	0.78	1.31
YANKEE_BB	1.06	0.43
CANADIAN_B	0.92	1.30
ENERGY_B	1.24	0.84
FINANCIAL_B	1.00	0.20
INDUST_B	1.02	0.81
TELE_B	1.03	0.99
YANKEE_B	1.34	0.58
CCC	1.34	0.61
SWAP_5	1.04	1.23

Mortgage spread volatility

The mortgage spread volatility forecasts perform somewhat less well than the corporate spread forecasts. In the pre-crash period, the model over-forecasts mortgage spread risk by about 35%, while in the post-crash period it has under-forecast spread risk by the same amount [see Table 6]. The earlier bias was due largely to the aftereffects of the 1994 mortgage market volatility, while the later bias seems to be the result of steadily increasing mortgage spread volatility since the 1998 crash (as is visible in Figure 4).

Table 6. Pre-crash and post-crash bias statistics for mortgage spread factors, where shading indicates a bias statistic outside the 95% confidence interval shown in parentheses at the top

Factor	Jan 96–Jul 98 (1±0.26)	Dec 98–Jul 00 (1±0.32)
CONV_B	0.53	1.44
CONV_15	0.81	1.29
CONV_30	0.59	1.56
GNMAI_15	0.88	1.09
GNMAI_30	0.69	1.37

Tests of the specific risk and credit risk models

Asset-specific risk and issuer credit risk models are tested in the same manner as the common-factor model. That is, out-of-sample returns are compared to their model-predicted volatility (constructing standardized returns from their ratio) and then checked to confirm that the standard deviation of the ratio is close to 1.

In this case, rather than with factor returns, the concern is with asset-level returns after accounting for common-factor changes (that is, interest rate and sector-by-rating spread changes). These asset-level returns are measured as the residual change in OAS for each asset relative to the change in the common factor to which each is exposed.

Specific risk model

As described in "Model Structure and Estimation," specific risk forecasts for individual Treasury and agency issues and mortgage pools are derived using an empirical analysis of residual spread changes after accounting for common factors. This approach accounts at least in part for the observed pattern of asset-specific, liquidity-driven price distortions. Table 7 summarizes the test results for these three asset types, grouped by year and asset type.

Table 7. Bias statistics of residual OAS changes for mortgage passthroughs, agency, and Treasury bonds

	MBS	Agency	Treasury
1997	1.05	0.58	0.74
1998	1.18	1.43	1.07
1999	1.05	1.47	1.49
All	1.09	1.26	1.13

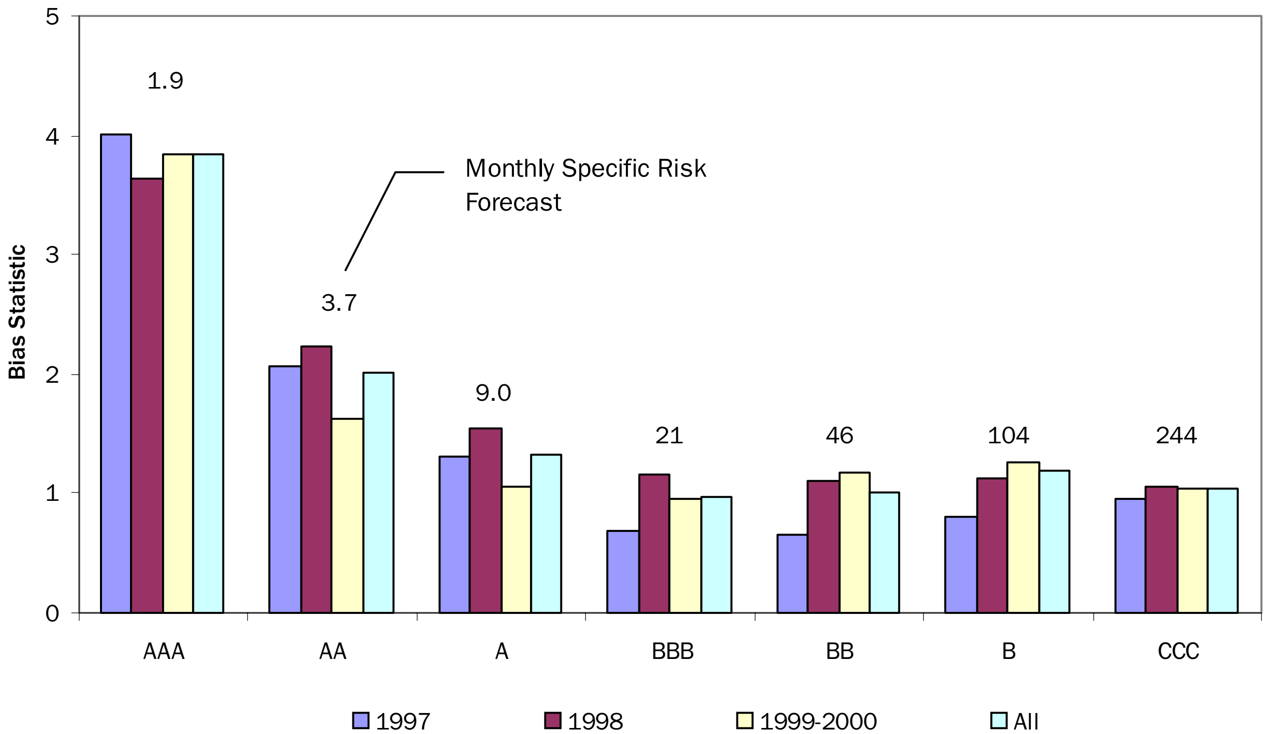
Two observations about the statistics are prominent:

- The bias statistic is close to 1 in most periods for all three asset classes. Mortgage pool residual spread volatility, in particular, appears to be stable and well forecast by the model.
- The values for Treasury and agency bonds are well outside the expected range under the assumption of independent specific returns in each period. However, the magnitudes of the residual spread volatilities are relatively small, as is the case with corporate AA and AAA bonds.

Credit risk model

Figure 9 shows the average bias statistic for individual corporate and non-U.S. sovereign bonds by rating and period. Each bar gives the standard deviation of the residual OAS changes divided by the credit migration forecast for all bonds with a given rating over a given period.

Figure 9. Bias statistics for the transition-matrix-based credit risk model



For bonds rated A and below, the forecasts of the credit migration model have been accurate over the sample periods, that is, the standard deviations are close to 1. This is consistent for volatility forecasts from the lowest (A) to the highest (CCC), spanning a range with a factor of almost 30. For issuers with the highest credit ratings, however, the credit migration risk forecasts are underpredicting residual volatility.

The underprediction of residual volatility for high credit issuers could be an artifact of our use of vendor-supplied prices, which are delivered as pricing matrices, rather than as transaction prices. Pricing discrepancies between our valuation methodology and that of our vendors (for example, in the construction of yield curves or in OAS model assumptions) could appear as excess residual returns, which would be most noticeable for bonds with low volatility.

It is also possible that the observed residual OAS changes for AA and AAA issuers are, in fact, larger than predicted by the model, possibly due to liquidity effects where particular issues become slightly rich or cheap due to trading behavior. However, even if this phenomenon exists, it seems unlikely that it would be observable from our data, especially given that we are using mostly matrix prices. Whatever the cause, the magnitude of the volatilities is small, such that the observed residual OAS volatility of AA and AAA bonds is approximately 7 to 8 basis points per month.

Summary

The U.S. fixed income risk model integrates common factor models of interest rate risk and spread risk for the taxable and tax-exempt markets with models for issuer-specific risk and issuer credit migration risk. The model covers major sectors of the Treasury, agency, corporate, sovereign, supranational, mortgage, and municipal bond markets.

The design of the model provides:

- Good market detail without introducing an excessive number of factors (which would give spurious correlations)
- Rapid response to changing market conditions, balanced against the need for sufficient data history to make accurate forecasts

Barra continuously monitors the model performance and looks for opportunities for enhancements.

Appendix

Factor structure motivation

Two technical questions arose in determining the factor structure of the model:

- Is the sector breakdown fine enough to capture the statistically discernible market structure?
- Should rating and sector be treated as separate factors, or should they be combined (as in Barra's current USFI model) in a sector-by-rating matrix?

Sector breakdown

The first question was answered by starting from a finer model (with 16 sectors) and looking for statistical evidence that some of the sectors had consistently high correlations in most (or all) rating categories. This detailed sector approach broke out the named subsectors from within the Financial, Yankee, Canadian, and Utility sectors.

The spread change series [see below] was estimated for each refined sector and rating group, and then their correlations were compared within each rating category. In other words, correlations were compared among all AAs, all As, all BBBs, and all BBs. AAAs and B and below were excluded from this comparison, because the sample contained too few bonds with those ratings.

Sample periods that both included and excluded the fall 1998 post-Russian-default crisis were used. A search was conducted for consistently high correlations among sectors across rating groups.

High correlations were found in a pattern that led to the final sector breakdown. That is, Canadian corporates are highly correlated with government and provincial bonds, Yankee corporates and sovereigns are highly correlated (though they were only slightly correlated with supranationals), as are utilities and financial issues. Thus, 9 sectors were retained from the original 16.

Separate factors versus matrix

The second question was asked because the previous U.S. Fixed Income risk model (the Barra B2 model) took the approach of treating sector and rating separately, rather than combining them in a matrix. There was a separate risk factor associated with each rating category below AAA, and another risk factor for each sector. So an A-rated industrial bond would be sensitive to both the A-rating spread factor and the industrial sector spread factor. The B2 method is called a "Rating + Sector" or an "R+S" model, and the new method is called a "Rating × Sector" or an "R×S" model. (In each case, inserting into the formula the number of rating and sector categories gives the number of factors in the model.)

Econometricians are inclined to prefer the B2 approach, because it keeps the number of explanatory factors to a minimum. (The 61 spread factors would be reduced to just 21 factors in the B2 approach.) The B2 approach, however, requires that there be no "cross-terms" between the rating and sector factors. For example, the difference in spread changes between A-rated industrial bonds and B-rated industrial bonds, and the corresponding difference in spread changes between Yankee A-rated and Yankee B-rated bonds should be about the same over any period.

This works well for investment-grade bonds, where all changes in bond spreads can be described as due to a combination of a sector spread change and a rating spread change. The B2 model covered only investment-grade bonds, so the R+S methodology was acceptable, but the model breaks down badly when it is used for bonds with ratings below BBB.

A test of explanatory power called an "F-test" was used to compare the two methods. Given a model of some data and a proposed additional explanatory factor, the F-test measures whether the added factor gives a statistically significant improvement in the model. The R+S model can be viewed as a constrained embedding within the R×S model, so an application of the F-test to compare the two can tell whether at least some of the additional factors in the R×S model are significant.

The F-test demonstrated that the added explanatory factors are necessary to accurately model the full factor structure of the U.S. non-government bond market. Particularly in a high volatility period, the R×S model substantially outperforms the R+S model. For instance, in August 1998, the fraction of explained variance in bond spreads (R^2) of the R×S model was 81%, while for the R+S model it was just 47%. The R×S model has more factors, so a higher R^2 is expected; however, the F-test in this case demonstrates with very high confidence that at least some of the additional factors add real explanatory power to the model.

Forecasting the one-month credit transition matrix

By its definition, the transition rate matrix Λ must have certain properties:

- The sum of each column must be 0. That is, all transitions out of one rating state must be accounted for by transitions into another rating state.
- The off-diagonal entries are positive or 0, while the diagonal entries are 0 or negative. The first part of this condition means that the rate of transition from one state into a different state cannot be negative. The second part of this condition is an implication of the previous condition.

These specified conditions on Λ are purely mathematical. Inspection of the entries for rows AA and A and column CCC of Table 8 motivate an additional financial consideration. The observed rate of transitions from AA to CCC is 0.02% per year, while the rate from A to CCC is 0.01% per year. Clearly, the transition rate from a lower rating to CCC should be greater than that from a higher rating.[16] As historical observations, there is nothing unreasonable about the numbers; they reflect random fluctuations in the very small number of cases of a highly rated issuer suffering a very large credit event over a one-year period. This fluctuation should not be enshrined in the forecasts, however.

The procedure for making use of the raw S&P transition matrix is then:

1. Find the corresponding raw rate matrix $\Lambda_{raw}=log(M)/T$ where $T=1$ year.
2. Estimate the "closest" model rate matrix Λ satisfying the required mathematical and financial conditions. This is accomplished by minimizing an objective function that penalizes deviation of the model matrix from the observed one while respecting the constraints.

Given the model rate matrix Λ, the model transition matrix M for any horizon can then be found. The risk model is designed for forecasts over a one-month horizon, so the one-month transition matrix is calculated: $M(1\ month)=exp(\Lambda/12)$. This matrix, shown below in Table 8, is then used to forecast the return volatility due to issuer credit events.

Table 8. One-month transition matrix derived from the S&P matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	99.36	0.61	0.02	0.01	0.00	0.00	0.00	0.00
AA	0.05	99.28	0.62	0.04	0.00	0.01	0.00	0.00
A	0.04	0.21	99.24	0.45	0.04	0.02	0.00	0.00
BBB	0.02	0.02	0.49	98.94	0.45	0.06	0.01	0.02
BB	0.02	0.01	0.03	0.67	98.34	0.78	0.11	0.06
B	0.02	0.01	0.03	0.13	0.58	98.36	0.44	0.44
CCC	0.02	0.00	0.03	0.13	0.15	1.17	96.22	2.28

1. This model, released in 1999, is the successor to the Barra B2 model, first released in 1987.

2. In the parametric approach, the specific risk and credit risk terms enter the risk calculation in very similar fashion, with credit risk reduced to a return variance. In a Monte Carlo scenario framework, they would show up with very different distributions, because the credit migration distribution is highly non-normal.

3. Uncertainty about future volatility and prepayments also plays a role for some assets. In the model, these uncertainties are captured through their influence upon market spreads.

4. See "Factor structure motivation" on page 29 for a discussion of the empirical basis for this specification. In broad terms, the main criteria in selecting the factors are explanatory power, intuitiveness, and market practice.

5. The simple formula given here is not precisely the one used. The Treasury spot rate factor return is actually calculated as the change between [*the one-month forward rate to the specified maturity at the earlier date*] and [*the spot rate to the same calendar date at the later date*]. That is, it is the change in a fixed calendar interval of the forward curve. For example, the "two-year spot rate change" is actually the change from [*the 1-month rate 23 months forward*] to [*the 1-year 11-month spot rate at the later date*].

6. Available online at http://www.barra.com/Newsletter/nl164/TNCNL164.asp.

7. Bonds for which no information is available from the prior month end are discarded, and outliers are screened. If, after this step, fewer than five bonds are in the factor group, the factor spread change is proxied by the average spread change for all bonds with the same rating. This is relevant primarily for some AAA-, BB-, and B-rated sectors, and for supranationals rated below AA.

8. Pools in the mortgage market are priced based upon generic characteristics (such as agency, program, coupon, and age), and the generics are in turn priced relative to TBA prices, so the methodology captures the primary determinants of pool pricing. The TBA selection methodology is motivated by the fact that TBA pricing is analogous to the "cheapest to deliver" pricing of bond futures: the TBA generic is the least valuable one with specified issuer, program type, and coupon.

9. Barra's muni valuation model takes account of the pre-refunding option and the call option. Yields for maturities of 12 years and longer are assumed to be for bonds callable in 10 years at 102, declining to par in year 12.

10. Implicit in this calculation is the assumption that the factor changes are normally distributed and serially independent.

11. Details of the method can be found in R.A. Johnson and D.W. Wichern, *Applied Multivariate Statistical Analysis*, Prentice Hall, 1992.

12. The spread change may occur before the rating change, if market participants are aware of credit events earlier than the rating agencies, but that does not matter to the argument. The rating transition matrix tells us the rate at which credit events of a given magnitude are occurring, even if the rating agencies' assessment of the credit events lags by some months.

13. Or, for that matter, in S&P's database: see the discussion in "Forecasting the one-month credit transition matrix" on page 31.

14. In this representation, a portfolio whose weighted-average distribution of ratings at the start of a month is described by the vector x_{it} has a month-end rating probability distribution of $\sum_i M_{fi} x_i$.

15. Before making these calculations, the "N.R." column is eliminated. To deal with transitions to non-rated status, the rate matrix Λ is calculated that would have been obtained had those transitions been excluded from the sample. They are excluded because no useful inference can be made about the implications for the value of a bond from an issuer whose debt becomes unrated.

16. Note that the same assumption cannot be made about the rate of transitions from one initial rating to two final ones. That is, there is no strong reason to assume that the rate of transitions from AA to B should be smaller than from AA to BB, even though BB is "closer" to AA. The reason is that the B category may be, in some sense, larger than BB, so that even though it is farther from AA, a credit event may be likelier to land an initially AA issuer in B than in BB.

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