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IN SEARCH OF A MODIGLIANI–MILLER ECONOMY

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The Modigliani–Miller theorem describes conditions under which the value of a firm is independent of its leverage ratio. It is one of the cornerstones of finance. A history of this result along with a modern perspective on its derivation is given in Rubinstein (2003), Journal of Investment Management **1**(2). *We extend this history by examining the relationship between the Modigliani–Miller theorem and quantitative models of credit risk. In the first part of the paper, we sort out the role of the Modigliani–Miller theorem and Merton's classical structural model. This material may be familiar to some readers. Subsequently, we explore the relationship between the Modigliani–Miller theorem and* I^2*, which is a hybrid structural-reduced form model based on incomplete information, Goldberg (2004),* Risk **17**(1), 515–518. *The* I^2 *model is not consistent with the Modigliani–Miller theorem. It provides a new way to measure the deviation of real markets from the idealized markets in which the Modigliani–Miller theorem holds.*

1 Introduction

A public firm raises money to finance its operations by issuing equity and debt. One of the pillars of modern finance is that, apart from tax considerations, it does not matter to a firm's investors how the firm raises money. In other words, if taxes are ignored, the value of the firm is unaffected by its capital structure. This is known as the *Modigliani–Miller theorem*.

The Modigliani–Miller theorem, denoted by M^2, has been the subject of enormous controversy. Aspects of this are examined in Rubinstein (2003), which points out that the statement and proof of a Modigliani–Miller type result can be found in Williams (1938). Not only does Williams's result predate the famous paper of Modigliani and Miller (1958) by 20 years, it has a broader reach. For example, in their "no arbitrage" argument, Modigliani and Miller (1958) compute the present value of firm debt by discounting at a risk-free rate, thereby

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neglecting firms that are subject to default. The argument in Williams (1938) does not suffer from this constraint.

Rubinstein (2003) concludes by looking backward to the Modigliani–Miller theorem from the perspective of modern finance. He identifies a minimal set of axioms required for M^2 to hold. These are

1. There are no riskless arbitrage opportunities.
2. Operating income (from assets) is not affected by capital structure.
3. The proportion of operating income that is jointly allocated to stocks and bonds is not affected by the firm's capital structure.
4. The present value function (the economy wide state prices) is not affected by capital structure.

In order to derive the Modigliani–Miller theorem from these axioms, think of a firm as a portfolio composed of equity and debt. Then, the value of the firm is the expected present value of the future operating income it generates and M^2 can be restated as follows: the expected present value of a firm's future operating income is independent of the *leverage ratio* of debt to firm value.

The proof goes as follows. Let Ω be a set whose elements represent the states of nature. The physical probability, denoted by P, gives the likelihood of any observable event. The first "no arbitrage" axiom guarantees the existence of a pricing kernel, or *present value function* V. This is a stochastic process whose value $V_T(\omega)$ gives the present value of one dollar at time T in state ω. The present value of the random payoff $X(\omega)$ at time T is given by the average

$$E^P[V_T X] \tag{1}$$

of $V_T(\omega) \cdot X(\omega)$ over all possible states of the world ω, weighted with their physical likelihoods $P(\omega)$. If X is the value of the firm at T, then the expectation (1) gives its fair, arbitrage-free value today. The second and third axioms imply that X does not depend on the leverage ratio and the fourth implies that V does not depend on the leverage ratio.

The four axioms cited above describe an idealized economy. Rubinstein (2003) remarks that the Modigliani–Miller theorem serves less as a statement that the leverage ratio is irrelevant to firm value than as a benchmark from which to measure the ways in which leverage ratio affects firm value. He adds that each of the axioms mentioned above has generated "its own virtual cottage industry of academic research."

Cottage industries notwithstanding, the Modigliani-Miller theorem is consistent with many quantitative credit models in common use today.

2 M^2 and Merton

Merton (1974) describes the first modern quantitative credit model. His stated goal was to find a fair price for corporate debt. In Merton's model, a firm has a simple capital structure consisting of equity and a single zero coupon bond. Default occurs only at bond maturity and only if the value of the firm is below the face value of the debt.

The Merton model is based on a so-called "perfect" market, which resembles the ideal world described by Rubinstein (2003). There are no transaction costs, no taxes, no bankruptcy costs and no stock repurchases. New debt cannot be issued until the old debt is retired. In Merton's perfect market, equity S is a European call option on firm value X, which is assumed to be a geometric Brownian motion. The strike price K is the face value of the debt and the option can be exercised only at debt maturity T. The formulae in Black and Scholes (1973) can be used to estimate the parameters of the firm value process from the current equity value and volatility, and the face value and maturity of the debt. The present value of the debt is the difference between firm value and equity value. It can be expressed as the present value of its face value less the value of a European put option U on the firm X with strike K and maturity T as a consequence of the put–call parity relation:

$$X_0 = S_0(K, T) + Ke^{-rT} - U_0(K, T)$$

The initial set of assumptions in Merton (1974) includes the Modigliani–Miller theorem. At that time, there was disagreement about whether or not M^2 holds if bankruptcy is allowed. One source of the confusion is illustrated by the following example. Imagine two firms whose assets consist entirely of cash. Suppose that the amounts of the cash held by the two firms are the same and the investment policies of the two firms are the same, but that one firm raised its cash by issuing equity and the other by borrowing money. Do the firms have equal value? They do not. There is a significant chance that the levered firm will face bankruptcy and, therefore, incur charges that the unlevered firm does not have to worry about. Therefore, while M^2 might apply in the face of an abstract bankruptcy, it does not hold in the face of a realistic bankruptcy, which tends to be expensive. As noted above, Merton (1974) assumes bankruptcy costs nothing.

Curiously, M^2 is not explicitly mentioned in the derivation of the fair value of the debt. A clue to its role is provided in Merton (1974, Section 5), which derives M^2 from the other model assumptions. As summarized by Miller (1988), Merton reasons that although both the value of debt and equity are functions of the firm's leverage through K, their sum X_0 is independent of leverage. Logically, if this reasoning is correct, then the assumption of M^2 in Merton (1974) is redundant. However, this reasoning left some unconvinced; see, for example, Long (1974a, b). In response to criticism, Merton (1977) gives a second proof of M^2 from a set of axioms that is similar to the set in Merton (1974). This did not serve to diminish the confusion surrounding the Modigliani–Miller theorem. It raised more questions. Are Merton's first set of axioms equivalent to his second set? What are the minimial set of assumptions required to derive Merton's model? What is the exact relationship between Merton's model and M^2?

From the present perspective, a quick path to a definite statement is to observe that the four axioms cited above are implied by Merton's original assumptions minus M^2. Logically, this allows us to conclude that if all of the original assumptions in Merton (1974) except M^2 are, in fact, required to derive the debt formula and if M^2 fails, then the debt formula must be modified.

The first three axioms are virtually embedded in Merton's assumptions. The fourth axiom requires some explanation. Merton's model is limited in the sense that there is room for only one present value function. Thus, the fourth axiom is trivially satisfied. A change in capital structure cannot change the present value function because there is nothing to change it to.

3 More about present value functions

All other things being equal, most investors prefer less risk to more. Consequently, market prices include compensation for the risk that cannot be diversified away. This *risk premium* is realized as an increase in expected return on investment. It generates a *market-implied* or martingale probability model in which the likelihood of an event is the physical likelihood adjusted by the risk premium.

Here is an example. Risk-averse investors will not trade a sure million dollars for a fifty–fifty bet on two million or nothing, even though both investments have an expected value of one million dollars. However, if the odds are sufficiently skewed toward the two million, the risky deal becomes more attractive. Investors may disagree about how heavily skewed the odds must be before they trade. Each view corresponds to a price for the risky deal that is lower than one million dollars. The market average price implies the martingale probability model.

It is well known that the connection between the physical probability model P and the one implied by market prices is contained in the present value function V, which can be decomposed into economically meaningful factors:

$$V_T = B_T \cdot Z_T \tag{2}$$

The variable $B_T(\omega)$ is a *discount factor* that represents the time value of money in state ω. For example, if the continuously compounded risk-free rate is a constant r, then $B_T = \exp(-rT)$ in all states. The variable $Z_T(\omega)$ is called the *density*. It represents the influence of relative supply and demand of wealth in state ω. It is closely related to the risk premium demanded by investors, as we discuss below.

In Merton's model, there is a single source of uncertainty, which is a standard Brownian motion W. At time T the value of the firm is given by

$$X_T = X_0 \exp\left(\left(\mu - \tfrac{1}{2}\sigma^2\right) T + \sigma W_T\right) \tag{3}$$

where μ is the firm growth rate, σ the volatility of firm value, and X_0 the current value of the firm. This means that firm value is log-normally distributed in the future. In this situation, the density Z is uniquely determined through market prices as

$$Z_T = \exp\left(-\lambda W_T - \tfrac{1}{2}\lambda^2 T\right) \tag{4}$$

where the constant $\lambda = (\mu - r)/\sigma$ describes the excess return of the firm over the risk-free return per unit of firm risk, measured in terms of volatility. This quantity is called the market price of risk, or the *risk premium*. If the market is risk averse, then λ is positive. A risk-averse market places more weight on unfavorable states of nature than reality dictates. For example, market-implied probabilities of default are higher than physical default probabilities based on P.

The familiar process (4) that represents the density in Merton's model is an example of a large class of processes that can serve as densities for models of financial markets.

4 M^2 and I^2

Merton's model is consistent with the Modigliani–Miller theorem. However, like M^2, the Merton model is not entirely consistent with real markets. For example, it cannot be fit to the empirically observed positive short term credit spreads, nor does it account for the abrupt drops in security prices that occur at default.

There are modern descendants of the Merton approach that give more realistic pictures of today's markets. One collection of these is the family of *incomplete information* structural models that explicitly incorporate the level and quality of information available to investors. Incomplete information models are developed in Duffie and Lando (2001), Giesecke (2001) and Çetin *et al.* (2002). They need not be consistent with M^2. We illustrate this with a particular example, denoted by I^2, which is described in Giesecke and Goldberg (2003a, b).

There are many similarities between I^2 and the Merton model. Both are structural in the sense that they are based on a model definition of default.[1] Both use a geometric Brownian motion to generate the uncertainty of firm value prior to default. However, unlike the Merton model, I^2 accounts for both the short term uncertainty of default and abrupt drops in security prices.

In I^2, firm value jumps at default. Equity becomes worthless and the bond value is diminished by bankruptcy costs. Thus, I^2 is not consistent with M^2 since the operating income depends explicitly on the leverage ratio. In other words, I^2 violates axiom 2.

Further, I^2 violates axiom 4. To see this, we examine the I^2 present value function $V = BZ$, which has a richer structure than the Merton present value function. The geometric Brownian motion underlying firm value is the only source of uncertainty in the Merton model. In I^2, there is a second, independent source of uncertainty arising from the economic assumption that investors cannot observe the value of the firm that triggers default. Therefore, the default time depends on two stochastic factors: firm value and the location of the default barrier. Correspondingly, the I^2 density Z is a product of two terms

$$\begin{aligned} Z_T = {} & \exp\left(-\alpha W_T - \tfrac{1}{2}\alpha^2 T\right) \\ & \times \exp(\log(1+\beta)N_T - \beta A_T) \end{aligned} \tag{5}$$

as is shown in Giesecke and Goldberg (2003b, Section 5).

The first factor in formula (5) is analogous to the Merton density. The quantity α adjusts the growth rate of the firm value process just as $\lambda = (\mu - r)/\sigma$ does in the Merton model. If the market is risk averse, then the growth rate is lowered. We interpret α as a premium for the "diffusive risk" in the firm value, which is represented by the Brownian motion W prior to default.

The second, "jump" term, which does not appear in the Merton density, requires more explanation. The variable N_T is an indicator of default: it is zero if the firm has not defaulted by time T and one if the firm has defaulted on or prior to time T. The process A_T is a continuous, non-downward "drift" that compensates for the fact that the probability of default is a non-decreasing function of term.

The jump term is effectively an adjustment to the timing risk of default. To see this, consider that risk-averse investors are more concerned about a near-term default than one in the distant future. Thus, the weight on near-term defaults in the market-implied probability model is greater than in the physical probability model P. The physical likelihood that a firm defaults before time T is just the P-probability of the set $\{\omega \mid N_T(\omega) = 1\}$. Analogously, the market implied likelihood is the size of this set under the market implied probability. Formula (5) can be used to show that the market implied likelihood is the physical likelihood modulated by a function of the coefficient β.

The first factor in formula (5) is a process with continuous paths. All the jumps are in the second term. It follows that $\log(1+\beta)$ can be interpreted as the premium for the "jump risk" in I^2. Driessen (2002) demonstrates empirically that the jump risk premium accounts for a significant portion of corporate bond returns.

In Giesecke and Goldberg (2003b, Section 5), it is shown that the assumption of no arbitrage is realized in the mathematical relationships among α, β, the recovery rate assumed by the market, and the coefficients of the price processes of traded securities. The market may be complete or incomplete, depending on which securities are traded. In either case, however, the price processes depend explicitly on the leverage ratio, so the premia α and β do as well. Thus, the density, and hence the present value function depends on leverage and axiom 4 does not hold.

5 A jumping off place

The modern perspective of Rubinstein (2003) elucidates the relationship between the Merton credit model and the Modigliani–Miller theorem. This perspective highlights the fact that these subtle propositions are "jumping off places" that belong to an ideal world. By modifying the minimal assumptions underlying M^2 and the Merton credit model in an orderly fashion and comparing output, we can gauge the impact of bankruptcy costs, incomplete information and other real world nuisances on the value of a firm and its debt. As indicated in Rubinstein (2003), there is an enormous literature dedicated to this type of analysis. The I^2 model can be useful in this regard since it gives a precise framework in which to measure the jump and diffusive risk premia as well as the market's recovery assumptions. The process of using I^2 as a measurement tool is begun in Giesecke and Goldberg (2003a, b). However, a full exposition of the results will have to await another opportunity.

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Note

[1] An alternative to the structural approach is the reduced form approach, which directly models the conditional default rate. The I^2 model can be represented as a generalized reduced form model although the Merton model cannot.

References

Black, F. and Scholes, M. (1973). "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* **81**, 81–98.

Çetin, U., Jarrow, R., Protter, P. and Yildirim, Y. (2002). "Modeling Credit Risk with Partial Information." Working Paper, Cornell University.

Driessen, J. (2002). "Is Default Event Risk Priced in Corporate Bonds." Working Paper, University of Amsterdam.

Duffie, D. and Lando, D. (2001). "Term Structures of Credit Spreads with Incomplete Accounting Information." *Econometrica* **69**(3), 633–664.

Giesecke, K. (2001). "Default and Information." Working Paper, Cornell University.

Giesecke, K. and Goldberg, L. (2003a). "Forecasting Default in the Face of Uncertainty." *Journal of Derivatives*, forthcoming.

Giesecke, K. and Goldberg, L. R. (2003b). "The Market Price of Credit Risk." Working Paper, Cornell University.

Goldberg, L. R. (2004). "Investing in Credit: How Good is your Information?" *Risk* **17**(1), S15–S18.

Long, J. B. (1974a). "Discussion." *Journal of Finance* **29**, 485–488.

Long, J. B. (1974b). "Stock Prices, Inflation and the Term Structure of Interest Rates." *Journal of Financial Eocnomics* **1**, 131–170.

Merton, R. (1977). "On the Pricing of Contingent Claims and Modigliani–Miller theorem." *Journal of Financial Economics* **5**, 241–249.

Merton, R. C. (1974). "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance* **29**, 449–470.

Miller, M. H. (1988). "The Modigliani–Miller Propositions after Thirty Years." *Journal of Economic Perspectives* **2**(4), 99–120.

Modigliani, F. and Miller M. H. (1958). "The Cost of Capital, Corporation Finance and the Theory of Investment." *American Economic Review* **48**(3), 261–297.

Rubinstein, M. (2003). "Great Moments in Financial Economics: II. Modigliani–Miller Theorem." *Journal of Investment Management* **1**(2), 7–13.

Williams, J. B. (1938). *The Theory of Investment Value*. Cambridge, MA: Harvard University Press.

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