RESEARCH INSIGHTS

The Barra Credit Series:

Empirical Credit Risk

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Abstract

We describe an empirically motivated model of credit risk based on a study of the relation between returns to corporate bonds, government bonds and equities. Examining 181,000 monthly return events spanning 6+ years, we find a clear systematic relationship between issuer credit quality, as measured by its bonds' yield spreads, and the attribution of its bonds' returns to interest rate changes and the issuer's equity return. Returns to high quality bond, with low yield spreads, are largely explained by interest rate changes, while returns to low quality bonds, with large spreads are primarily explained by the issuers' equity returns. Returns to bonds of intermediate credit quality are not significantly explained by either interest rate changes or equity returns, and appear to be attributable only to bond market specific factors. ("Explained" here does not imply causation, but merely dependence in a regression.) We also find evidence of an agency effect in the weaker correlations between bond returns and positive firm-specific equity returns than between bond returns and equity common-factor returns or between bond returns and negative firm-specific equity returns.

Using a heuristic model giving the regression coefficients of the bond return relationship in terms of the level of bond spreads, we describe an improved approach to modeling credit risk for corporate bonds, effectively accounting for correlations induced by market common factors.

1. Introduction

There has been great interest in recent years in obtaining improved quantitative understanding of credit risk. Most recently, portfolio shifts from equities to bonds, increased corporate bond issuance, the well-publicized defaults of several large issuers, and a surge in interest in credit derivatives have motivated a large amount of research and model development. The prototype for many of these models is the Merton (1974) model. Models tracing their lineage back to Merton's are referred to as "structural" models, because they take as a primary input information about a firm's capital structure. Alternative, more empirically motivated models are based on observed credit migration rates, or combine empirical data with information from structural models (e.g., RiskMetrics' CreditMetrics model).

A common application of standard credit risk models is forecasting of default probabilities —either from fundamental data or from transition probabilities. For a bank, holding illiquid, private, difficult to price loans, a default probability statistic may usefully function as a quantitative alternative to agency (e.g., Standard & Poor's, Moody's) credit rating. For any investor, default probability estimates may be a useful valuation measure, indicating which bonds in a portfolio are over or under priced relative to their default risk. More sophisticated versions of the Merton model, such as the VK model of Moody's/KMV and Barra's default probability model have become commercially successful.

However, a significant difficulty for many models of credit risk is to provide predictions of return correlations across issuers. These correlations are clearly of central importance to understanding risk in a portfolio context. Bond or issuer default probability estimates alone may provide a misleading picture of overall portfolio risk. Concentration in one sector (e.g., airlines) may result in a relatively large portfolio loss—even in the absence of many actual defaults—while a portfolio with good sector diversification may have the same overall distribution of default probabilities but a much lower probability of large losses. In principle, structural models should be capable of providing information about correlations of bond returns and default probabilities, derived from predictions of firm value correlations (such as those provided by Barra's equity models).

In practice, such applications have not had great success. Partly, this is because structural models do a poor job of fitting bond price and return data "out of the box" (Eom, Helwege and Huang (2002), Huang and Huang (2002)). In practice, they need to be calibrated to empirical default rates and bond spreads in order to be useful as market models. Once this has been done, such a model is no longer genuinely predictive, since its forecasts have been modified to fit historical or current observations. Instead, it can be thought of as providing a plausible basis for interpolating and extrapolating the calibration data, based on factors relevant (according to the model) to default rates and/or bond spreads.

Aside from the calibration issues, structural models are difficult to use in practice for a number of reasons:

- they require accounting data, such as leverage ratios and other details about capital structure that may be difficult to obtain or out of date;
- they require estimates for bondholder recovery rates in default;
- realistic capital structures present computational difficulties or require simplifying approximations;
- highly levered firms such as banks, insurance companies and other financial companies present complications due to the offsetting nature of their assets and liabilities.

An investor with mark-to-market portfolios holding public securities will probably be most interested in market returns rather than the specific event of default. Defaults do not generally happen "out of the blue", so by the time legal or de facto default has occurred, affected securities will have been marked down to prices reflecting the perceived likelihood of default and amount of recovery. For purposes of risk modeling—if our goal is to forecast the volatility of asset or portfolio returns—a calibrated structural model may be sufficient, but it is not necessary. If we can estimate the correlation between a bond and its issuer's equity by some other means, as well as forecast equity return volatility and correlations, then we do not need forecasts of either default probabilities or recovery rates, or their impact on bond value. We can thereby sidestep the difficulties with all the inputs and outputs from structural models, and focus on just those results required for predicting the volatility or distribution of bond returns.

In this paper, we describe a statistical model of the relationship between the return to a credit-risky bond and the return to the issuer's equity. We take market prices as a given, using them as the basis for forecasting bond risk in terms of interest rate, equity and residual spread factors. We refer to this approach as "empirical credit risk" (ECR). This approach is somewhat akin to that of reduced-form models of credit risk. But instead of seeking to *value* bonds based on credit migration and implied or historical default probabilities, we seek instead to explain their *returns*, taking a bond's value (equivalently, its spread over a default-free rate) as a measure of credit exposure.

A number of previous studies (Kwan (1996), Alexander, et. al., (2000), Collin-Dufresne, et.al. (2001), Hotchkiss & Ronen (2002), Treptow (2002) and others) have examined empirical bond-equity return relationships. This paper extends the literature in several directions. First, we demonstrate that a bond's spread relative to the Treasury bond curve provides an effective measure of the degree to which the bond's return will be correlated with Treasury bond returns (i.e., by changes in default-free interest rates) and with the return of the issuer's equity. Previous studies have found a weak relationship based on agency rating. From the standpoint of modeling risk, the main advantage of using bond spread is that it is a much more timely and responsive measure of perceived credit quality than agency rating.

Second, we examine a number of factors that may have additional influence on the return relationships, finding two clearly significant effects. Bond duration is found to increase the exposure of bonds to equities at intermediate levels of credit quality, though not for the most distressed issues. Most intriguingly, we find clear evidence that the bond-equity return linkage is significantly weaker for *positive equity specific return* than for other sources of equity return (common factors or negative issuer-specific events).[1] We tentatively ascribe this to agency effects where management acts to increase shareholder value at the expense of bondholders. On the other hand, we find no clear impact of equity volatility or sample period, and weak evidence of sector dependence. We also see no difference in the behavior of "natural" high yield bonds compared to fallen angels.

Third, we propose to use the empirical relationship between bond returns, interest rate changes and equity returns as the basis for a significantly improved approach to portfolio risk modeling. We show that, in combination with a factor model for the residuals of the attribution of corporate bond returns to equity and interest rates, we are able to account for anywhere from 55% to 90% of the variation in bond returns, depending on credit quality. This is a substantial improvement on a simpler model that ignores the attribution to equity, but also improves on a model that leaves out the residual credit spread factors, particularly for bonds of intermediate credit quality.

[1] Equity specific return is the component of a stock's return not explained by either passage of time (the risk-free rate) or equity market common factors.

The organization of this paper is as follows. Section 2 presents the ECR model structure, motivated in part by implications of the Merton model, and in part by practical experience. Section 3 describes the data. Bond price data is notoriously problematic, so we devote some attention to minimizing difficulties due to bad prices. Using monthly return data spanning more than six years for relatively liquid bonds found in the Merrill Lynch high grade and high yield indices, together with corresponding equity returns from a universe of 2000 issuers, we obtain a sample of roughly 180,000 linked return events (a bond return paired with an equity return) for analysis. Section 4 describes the overall results and a number of exploratory analyses of the data to determine the sources of variation in the ECR return relationships. Treating bond returns as dependent variables and interest rate changes and equity returns as independent variables, we find fitted exposures smoothly varying with and strongly dependent on bond spread. Section 5 discusses issues relevant to implementation of a risk model, including the addition of a spread risk model based on the residuals from the attribution of bonds returns to interest rates and equity returns. The bond-equity and bond-interest rate exposures are well captured by heuristic functional forms with reasonable limiting behavior governed by a small number of parameters. Section 6 concludes the paper.

2. Model Structure

We seek to model the return to a corporate bond in terms of the returns to government bonds — i.e., changes in default-free interest rates — and the return to the bond issuer's equity. We represent this relationship through the return decomposition

Equation 1 $$r^t_{B_{ISS}} = \beta_{IR} r^t_{B_{GOV}} + \beta_E r^t_{E_{ISS}} + \varepsilon^t_B$$

giving the excess return[2] $r^t_{B_{ISS}}$ to a corporate bond (denoted by the subscript B_{ISS}) in terms of the corresponding excess return $r^t_{B_{GOV}}$ to an equivalent government bond, the equity excess return $r^t_{E_{ISS}}$ and a residual. The coefficients β_{IR} and β_E measure the dependence of the bond return on interest rate changes and equity returns. The "equivalent government bond return" is derived by treating the bond as default-free, computing the implied interest rate factor exposures $-D_{B,i}$ ("durations", with conventional minus sign) and then summing the return contributions from interest rate factor changes obtained (by inversion of this relation) from actual government bond excess returns: $r^t_{B_{GOV}} = \sum_i (-D_{B,i}) r^t_{IR,i}$. The interest rate factors are given by approximate principal components of the term structure movements.[3] For high-grade bonds, the calculation of durations adjusts for the timing of

[2] Excess return is total return minus the risk free rate (but see section 3 for a more precise definition for bonds). The exact specification of excess return is inessential to the results.

[3] Barra's interest rate risk models incorporate three factors (approximate principal components of the covariance matrix of spot rates) responsible for most of the term structure variation in the 3-month to 30-year maturity range.

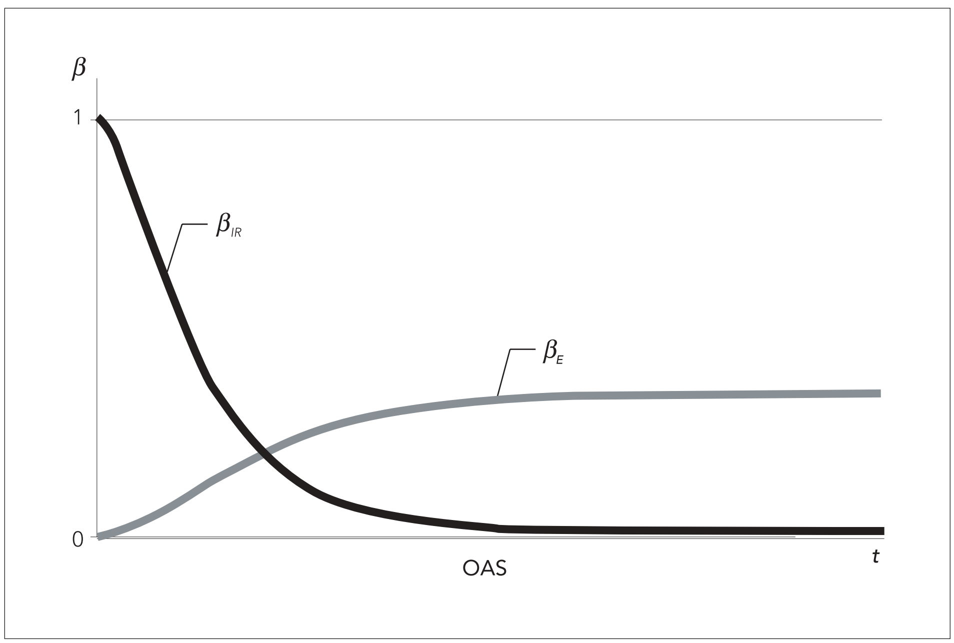
cashflows and other bond structural features, so that when the coefficients of Equation 1 are estimated using the government bonds originally used to find the interest rate changes (and taking $\beta_{E,} r_E^t = 0$, of course), we obtain β_{IR}.

Clearly, the coefficients β_{IR} and β_E will depend on characteristics of the bond or firm in question. Before looking at the data, we can anticipate some qualitative aspects of the relationship between bond prices, returns, interest rate changes and equity returns. We expect the option-adjusted spreads (OAS's) of bonds with very low default risk to be small.[4] Conversely, we expect that market OAS's of bonds with high default risk will be large. Accordingly, we anticipate that returns to low-OAS bonds will be primarily determined by changes in default-free interest rates, and nearly independent of equity returns, while returns to high-OAS bonds will be less sensitive to changes in default-free interest rates, and more sensitive to equity returns.

These criteria imply that β_{IR} and β_E should have roughly the behavior shown in Figure 1. β_{IR} is shown here as approaching zero at large OAS, but this is inessential—we expect only that it becomes substantially smaller than 1 for low credit quality bonds.

Figure 1

Schematic behavior expected for the relationship between bond, interest rate and equity returns as a function of OAS.



Although the Merton model is too simplistic to offer more than heuristic guidance, it is interesting to look at its predictions for the behavior of β_{IR} and β_E. These are shown in Figure 2 for one choice of model parameters. The form of the Merton model curves does not depend strongly on either the assumed equity volatility or the bond maturity.

4 The OAS is the amount by which the default free zero-coupon yield curve must be shifted in order that the bond valuation model reproduces the bond's market price. The valuation model captures contributions to bond value due to cashflow timing and embedded options, such as calls, puts and sinking funds, but assumes no default. (The valuation method is fairly standard. Details of the algorithm are available on request.) This definition generalizes the notion of yield spread relative to a default-free benchmark bond.

Curiously, the Merton model does not imply a single-valued relationship between bond OAS and β_{IR} and β_E. For small values of the OAS and fixed equity volatility, there are generally two corresponding values of the firm value and firm volatility, one near default with very low volatility, one far from default with higher volatility. Figure 2 shows only the branch corresponding to higher firm value and volatility.

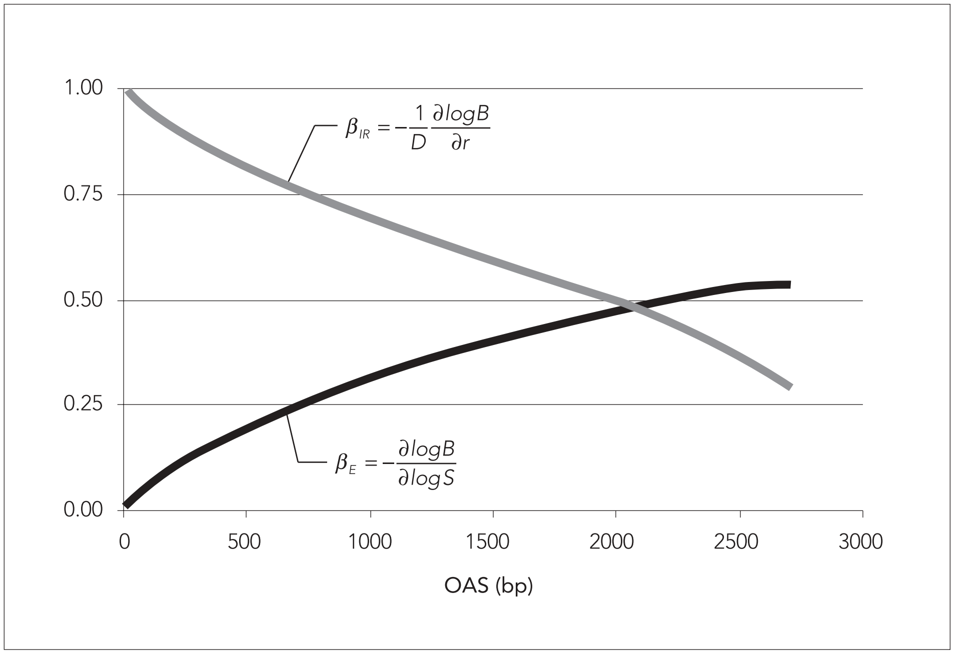


Figure 2

Merton model forms for β_{IR} and β_E as a function of OAS. The equity volatility is 100% annually. (At high OAS, this implies considerably lower firm value volatility.) The bond maturity is 10 years. *B* and *S* denote the bond and stock price and *r* denotes the interest rate. The shapes of these curves are not strongly sensitive to either the volatility or maturity assumption, but the largest realizable value of the OAS does depend strongly on the volatility. The relatively high value of 100% was chosen to span an interesting range of OAS's.

3. Data

The regression equation (Equation 1) involves three sets of returns. On the left side of the regression equation are bond excess returns. On the right side are equity excess returns, and "equivalent" government bond returns, or, in practice, interest rate factor returns and calculated exposures of the bonds to the interest rate factors. Our estimation universe, then, consists of a collection of bond and corresponding equity excess returns (typically more than one bond's return per equity return) pooled cross-sectionally and over time and the corresponding interest rate factor changes for each period in the sample. The interest rate exposures ($-D_{B,i}$) are calculated using a numerical model of bond value, calibrated to the market price of the bond, and taking the term structure of interest rates as an input.

The data come from several sources. Bond terms and conditions and prices are from the Reuters/EJV fixed income database. This database currently contains approximately 40,000 active USD corporate bonds. Many of these are small issues, medium term notes, or other illiquid securities. In order to improve the quality of the return data, we restrict analysis to those bonds found in a corporate benchmark of a major index provider. We chose the combination of the corporate component of the Merrill Lynch U.S. Domestic Master and the Merrill Lynch U.S. Domestic High Yield Cash Pay indices.[5] The first includes

[5] See http://www.research.ml.com/marketing/content/bond rules.pdf for further information.

fixed coupon bonds with maturity of at least one year, a minimum of $150 MM face value outstanding, and a minimum rating of BBB- from Standard and Poor's or Baa3 from Moody's. The second includes bonds of lower rating, the same maturity constraint, a minimum of $100 MM face value outstanding, and excludes deferred interest (such as pay-in-kind) bonds. By restricting our analysis to this estimation set, we can be confident that we are including only a relatively liquid portion of the bond universe.

Equity data are obtained from Bridge (Reuters) and IDC. We include the estimation universe from the Barra US equity risk model (USE3), consisting of 2000 equity issues, comprising the top 1500 public firms by market capitalization, plus an additional 500 firms chosen to fill thin sectors. The full dataset consists of bond and equity returns over the period beginning January 1996 and ending October 2002 (82 months).

We used monthly returns for the analysis, calculated from prices as of the last trading day of successive months. The regression equation (Equation 1) is a relationship between excess returns—that is, returns attributable to factors other than simply the passage of time. For a stock, this is just the total return less the risk-free rate (taken to be the 90-day TBill rate) that is, $r_{excess} = \frac{P_2 + C - P_1}{P_1} - r_{risk-free}$ where C is any cashflow received over the month (treated as being received at t_2). For bonds, rather than compute total return, then subtract the risk free rate to get the excess return, we compute a forward excess return for each bond over the period t_1 to t_2 as follows (ignoring here nuances related to settlement conventions in reported prices, which have negligible impact on our results):

- From P_1, the market price (including accrued interest) at t_1, calculate the spread OAS.
- Using this OAS, calculate a forward price P_1^{fwd} for the bond at t_2.
- Given the actual market price P_2 of the bond at t_2, the forward excess return is then $r_{excess} = \frac{P_2 - P_1^{fwd}}{P_1}$.

For straight bonds, this is equivalent to the total-minus-risk-free calculation. For bonds with embedded options, however, this method has the advantage of accounting for option time decay. As a check, we redid the analysis using conventionally calculated excess returns. (That is, the same method as used for equities.) The difference in results was entirely insignificant.

Bond and equity prices are central to our analysis. Equity prices are quite transparent. Given our focus on the largest capitalization issuers, we see no reason for concern about the quality of the equity return data used in the analysis. For bond data, the situation is entirely different. Whether obtained from Reuters/EJV or from another vendor, most reported bond prices are so-called matrix prices, derived from proprietary valuation models, rather than being actual traded prices. On any given day, no more than a thousand or so distinct bond issues trade, out of the tens of thousands outstanding, and almost all of these are unreported "over the counter" trades.[6] Pricing services use a mix of any traded prices they can get, trader indications (levels obtained on a non-tradable basis) and client feedback to calibrate their valuation models. As a result, prices or spreads supplied by pricing services (and for that matter, by traders giving indications) may deviate by anywhere from a few basis points to tens of points from a price at which a bond might actually change hands.

There are two sorts of potential problems with vendor prices that can affect our analysis. The most obvious is that a vendor will fail to capture new pricing relationships due to a change in valuation of a particular issuer, as might occur after a credit event. The obvious consequence is that the bond-equity return linkage will not be fully reflected in the data. (It would be very surprising to see the opposite effect—that a vendor pricing error could result in an apparent linkage between equity and bond prices where none actually exists.) A second problem is that, even if the vendor correctly tracks the valuation of an issuer over the longer term, there may be short-term mispricings that get corrected days or weeks after a market move. This will give rise to an apparent lag in the relation between equity and bond returns, an effect that has been reported in some past studies.

We have sought to minimize the impact of vendor pricing errors by using monthly returns, rather than higher frequency (daily or weekly) ones. This minimizes the impact of any delay by a vendor in repricing an issuer's bonds after a credit event. Only those cases where a delay moves the bond return associated to a credit event from the month of the event to a later month will affect the analysis. Such lags have the effect of reducing the apparent correlation between bond and equity returns.

As a further test of the impact of vendor prices, we reran the analysis using prices from an alternate (and perhaps more reliable) source, namely the Merrill Lynch Index group. Because our estimation universe was already based on the Merrill Lynch Domestic Corporate indexes, the only change was in the prices—not in the estimation universe. The results using the alternative prices were qualitatively the same, and quantitatively only slightly different from the results based on Reuters/EJV prices.

4. Analysis and Results

To build a risk model, we will eventually want to estimate smooth functional forms, qualitatively similar to the Merton model shapes, and giving reasonable asymptotic behavior for both β_{IR} and β_E at low and high OAS. But before estimating this heuristic model, we need to characterize the empirical behavior of the β's—we need to know what factors are responsible for variation in the dependence of bond returns on equity returns

[6] The NASD has recently initiated the TRACE system for reporting of institutional corporate bond trades. However, the number of bonds covered remains small, and the system has only been operating for a short time.

and interest rate changes, and we need to know what the smooth empirical functional forms should look like, at least in the range of OAS's where the results are not too noisy.

Starting with a regression of equation (Equation 1) on the aggregate data binned by OAS range, we then drill down to examine additional dimensions of possible variation. The statistical uncertainty on the regression β's is estimated by bootstrapping with 200 runs. All error bars in the figures and tables are one standard deviation uncertainty estimates from the bootstrap runs.

We take OAS as a basic market measure of credit quality that we expect to affect the regression coefficients. OAS is calculated relative to a default-free zero coupon yield curve fitted to Treasury bill, note and bond prices.[7] (An alternative would be to use the LIBOR/swap curve as the benchmark for OAS calculations, and to define the interest rate factor returns. A technical disadvantage would be that more bonds would have negative OAS's.)

We are primarily interested in identifying factors having an economically significant effect on the return relationship. We present the results of the regressions in tables and figures showing the estimated β's and their error bars from the bootstrap analysis. Although these results can be translated into statements about *t*-statistics on null hypotheses such as "$\beta_E = 0$" and "$\beta_{IR} = 0$", we have given *t*-statistics only for the aggregate regressions — not for the "drill down" analyses. The sizes of the error bars in the graphs provide clear visual evidence of the significance of the results. However, the *t*-statistics are easily obtained from the data in the tables, showing the estimated values and the one standard deviation error estimates.

Exploratory analysis was done on additional groupings of the data, by

- bond maturity (duration)
- projected equity volatility
- sample period
- equity return sign and common and specific equity returns
- sector
- fallen angels vs. speculative-at-issue

The first two tests are motivated by considerations from structural models. In particular, we expect that bond maturity and equity volatility should influence the exposures of bonds to equity. (The main effect of maturity on interest rate exposure for high quality bonds is already accounted for by the calculated factor exposures ($-D_{B,i}$).)

The other tests are of sources of variation not predicted by a structural approach. Grouping by sample period tests the stability of the empirical relationship over time. Grouping into positive and negative equity return sets tests, among other things, whether the general phenomenon of "down markets = higher correlations" holds in this area. We

[7] A brief description of the term structure model and estimation method is given in Bhansali and Goldberg (1997).

also did one further analysis based on a slight extension of the model of Equation 1, splitting the equity return into two components: a term due to the common-factor return, based on the USE3 equity model,[8] and the residual, or specific equity return. A substantial difference in the exposures of bond returns to the two components of equity return return would be hard to achieve in a structural model. Except for a few market factors, such as interest rates, structural models are directly sensitive to capital structure and firm value, but do not distinguish the effect of different sources of change in firm value.

Grouping by sector is of particular interest for financial firms—as noted above, such companies' debt is difficult to model in a structural framework. Finally, we were simply curious whether fallen angels behaved differently than bonds issued with speculative ratings. A positive result might indicate that fallen angels are followed differently by analysts—or by different analysts—than firms whose debt has never been investment grade.

For development of our "production" model, we use a bounded influence estimation method as a means of minimizing the impact of data outliers. However, for purposes of exposition we have derived the results in this paper using a conventional least squares regression. The quantitative results turn out to be fairly insensitive to the influence function used in the regression.

Aggregate Estimation Results

The results of the regressions on data binned by bond OAS are shown in Figure 3 and in the Appendix in Table 1. The graph shows three series of values: the interest rate β's, higher on the left; the equity β's, higher on the right, and the regression R^2, high on both the left and right, and falling to a minimum around 400 bp OAS. The one standard deviation error bars are too small to see for many of the data points, primarily at low OAS. Measured by the *t*-statistic, β_E is significantly different from 0 at all OAS levels, even the 0-50 bp range. However, equity return contributes substantially to explaining bond return only once the OAS exceeds about 200 bp. From Table 1 we see that the interest rate exposure, β_{IR}, is not significantly different from 0 for the OAS > 700 bp bins, well into high-yield territory. Correspondingly, in Figure 3 the error bars become too large to fit on the graph.

One notable feature of this graph is the minimum of the R^2 in the neighborhood of the investment-grade/high-yield boundary. For bonds with OAS between about 250 and 440 basis points, the model R^2 reaches a minimum of about 10%. The implication of this low R^2 is that neither interest rate changes, as measured by government bond returns, nor changes in firm value, as measured by equity returns, are explaining much of the bonds' returns.

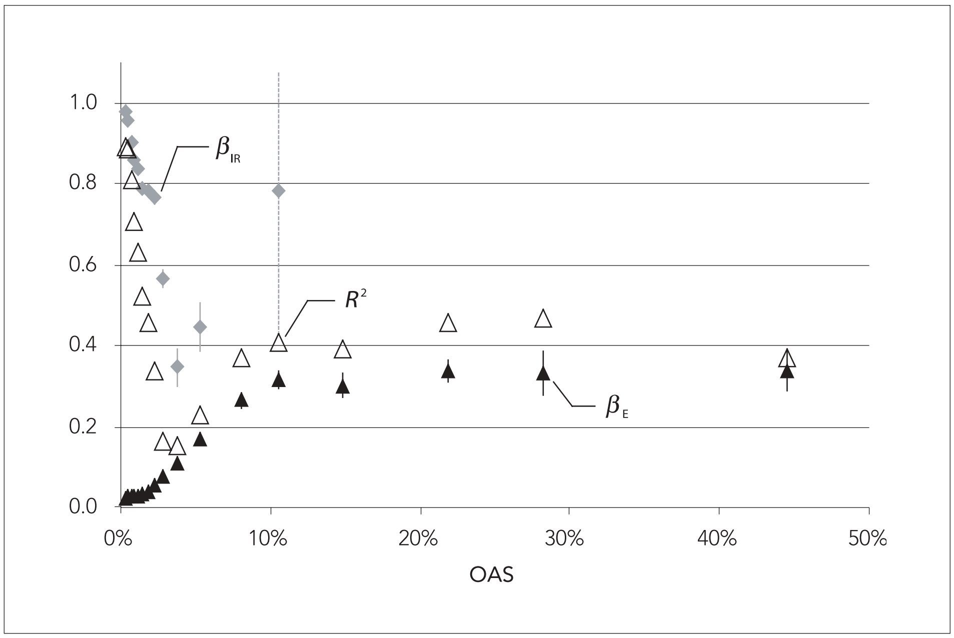
This result is consistent with the finding of Collin-Dufresne, Goldstein and Martin (2001).

[8] An overview with links to further details may be found at http://www.barra.com/support/equity_models/#us

They built several models of bond returns in terms of a variety of factors arising in structural models, such as changes in leverage and expected volatility. Although they find that some of these factors provide a statistically significant explanation of bond returns, they are unable to find structural proxies for default risk to explain more than a fairly small fraction of the returns. These authors studied the residuals from their models of bond returns, finding that they were at least partly due to one or more common factors not attributable to any plausible structural model. In other words, there are bond market "spread" factors, unrelated to identified sources of variation in firm structure or value, making a substantial contribution to bond total return. These factors must be taken into account by any reasonably complete model of bond risk.

Figure 3

Results of estimation of Equation 1 on bond and equity data binned by OAS, over the analysis period 1/1996 to 10/2002. Note that some of the β_{IR} estimates are off the scale, though portions of the error bars are still visible. Data are in Table 1.



A further interesting observation is the rapid drop in β_{IR} from near 1 for the lowest OAS bin to below 0.5 for bonds having OAS's above 350 bp, and for bonds with OAS above 700 bp, the estimate of β_{IR} is not significantly different from 0. The highest credit-quality bonds have interest rate sensitivity nearly equal to that of Treasury bonds with the same cashflows. On the other hand, interest rates explain essentially none of the returns of low credit-quality bonds. This is consistent with the market "lore," that duration overstates the sensitivity of lower-grade bonds to interest rates.

We now turn to examination of various additional factors potentially affecting the relationship between bond, equity and interest rate returns. Briefly summarizing the results:

- None of the examined factors strongly affect the estimates of the interest rate exposure coefficient β_{IR}. Interest rate exposure depends on bond characteristics as measured by its calculated exposures ($-D_{B,i}$) and its OAS, and not on any other factor examined.
- For given OAS in the intermediate range of 100–1000 bp, β_E is moderately increasing in

bond maturity or duration. One way of interpreting this result is that, to some degree, firm-specific news is absorbed into bond prices as a change in bond spread across maturities. Were this exactly true, with uniform spread change independent of maturity, we would find β_E to be proportional to duration. We find weaker dependence than this, implying that spreads of shorter term bonds change more than those of longer term bonds for a given equity return.

- β_E is not significantly sensitive to equity volatility, sample period, sector, or a firm's status as a "fallen angel".
- β_E depends strongly on the sign of the equity return, with negative return events having a much larger effect on bond return than positive return events. By splitting the equity term in Equation 1 into two components, we find that this effect is confined to firm-specific returns, and is not visible in market-wide (common-factor) returns.

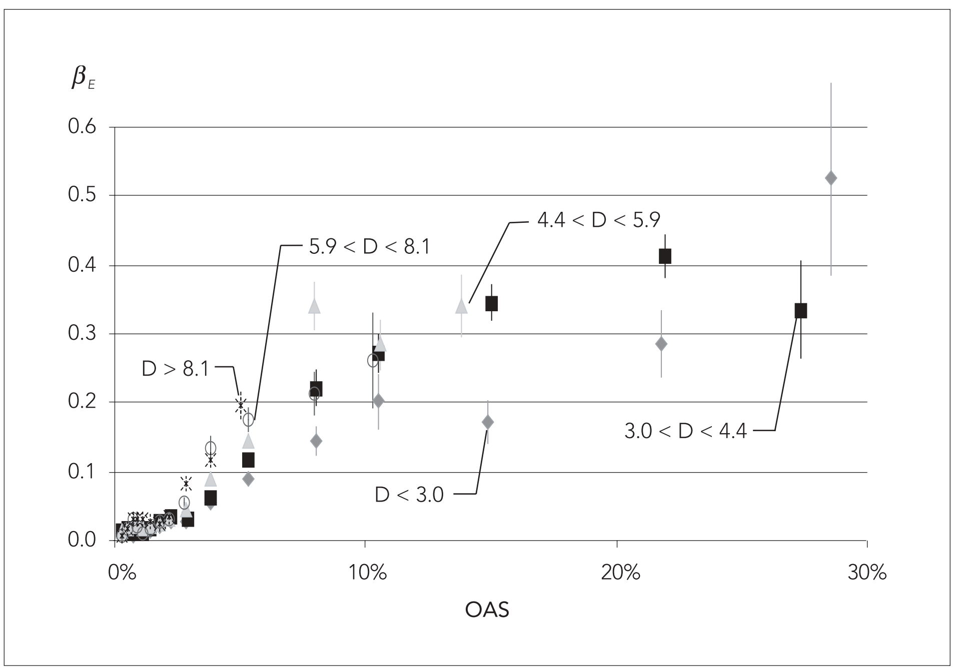
Bond Maturity

The longer a bond's time to maturity, the greater the chance of the issuer's default over its life. This trivial observation doesn't translate in any simple way to a specific relationship between OAS and interest rate or equity exposure, but it does suggest that the dependence is worth investigating.

To examine the dependence on maturity, we further bin the data. Rather than grouping by time to maturity, we used effective duration.[9] This has the benefit of taking account of the effect of embedded options, which is both important and calculable for high-grade bonds.

Figure 4

β_E estimates from regressions binned by OAS and effective duration. The duration ranges were chosen to have approximately the same number of returns in each range. The duration ranges are shown in the labels.

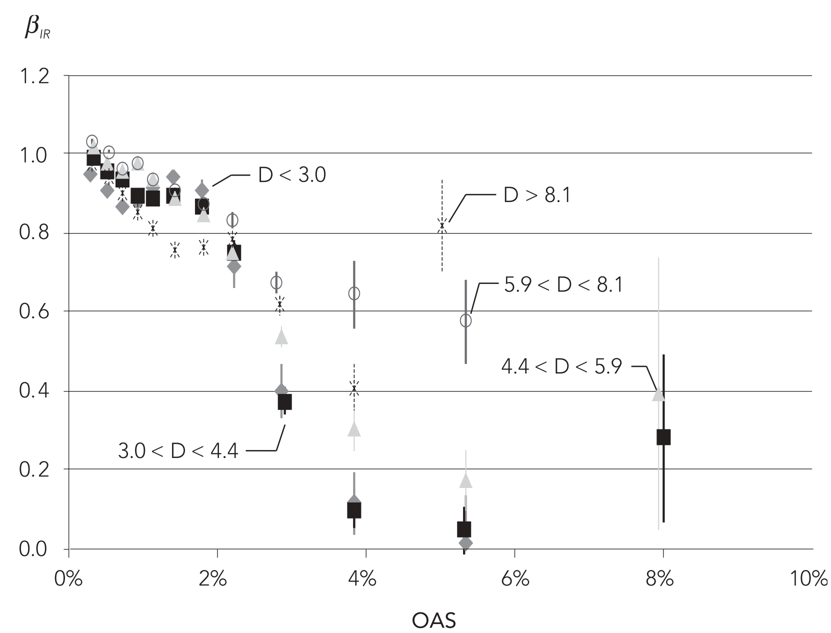


[9] The effective duration of a bond, like the factor durations of Equation 1, minus the elasticity of the bond's model value with respect to a change in the term structure—in this case, a uniform shift of the curve.

Figures 4 and 5 show the duration-binned estimates of β_E and β_{IR}. It is fairly apparent from Figure 4 that the rate of increase of β_E with OAS grows systematically from low to high duration. On the other hand, the maximum β_E of about 0.4 is not visibly different from low to high duration. Using a heuristic form for β_E (described in Section 5) we find that, for fixed OAS, the dependence of β_E on duration is increasing, but less than proportional. On the other hand, examining Figure 5, one sees that the dependence of β_{IR} on OAS does not vary significantly with duration.

Figure 5

β_{IR} estimates from regressions binned by OAS and effective duration.



Equity Volatility

The second test was of dependence of β_E and β_{IR} on equity volatility. The Barra USE3 risk model provides monthly forecasts of asset-level return volatility.[10] We use this forecast to bin the data into three groups: in each month, the groups comprise the lowest volatility quartile, the midrange half, and the highest volatility quartile. These three groups are then aggregated over all dates. The results are shown in Figures 6 and 7 and in the Appendix in Table 3. There is a hint from the analysis that the lowest volatility cohort has higher β_E at the highest OAS ranges (>1200 bp) than both the middle and highest volatility groups. However, the error bars on the estimates of β_E in this range are large. Moreover, this pattern does not appear in the intermediate OAS ranges (250 bp < OAS < 1200 bp),

[10] The USE3 model's predictions are quite accurate. For the roughly 1700 stocks continuously in the estimation universe over the period, the correlation between the model volatility forecast of 1/1/2001 with the subsequent realized 12-month volatility was 0.8. The rank correlation between forecast and realized volatilities was even higher, at 0.84. (The analysis here depends only on rank order of volatility forecast.)

where the errors are smaller, nor in the relation between the middle half and upper quartile volatility groups.

We see no evidence of any difference in β_{IR} across the three cohorts.

Figure 6

β_E estimates from regressions binned by OAS and forecasted equity volatility

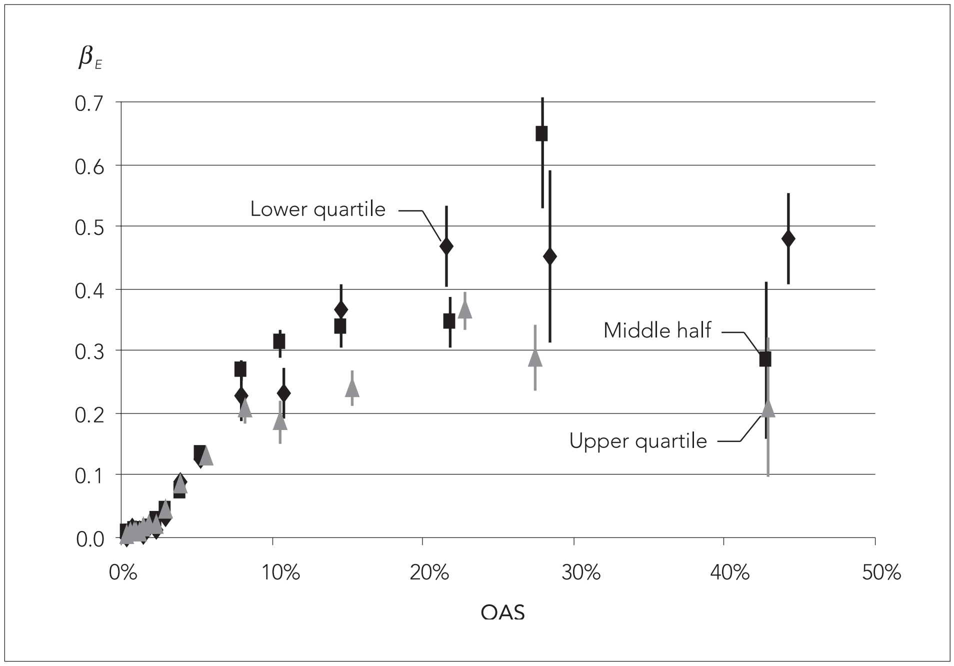
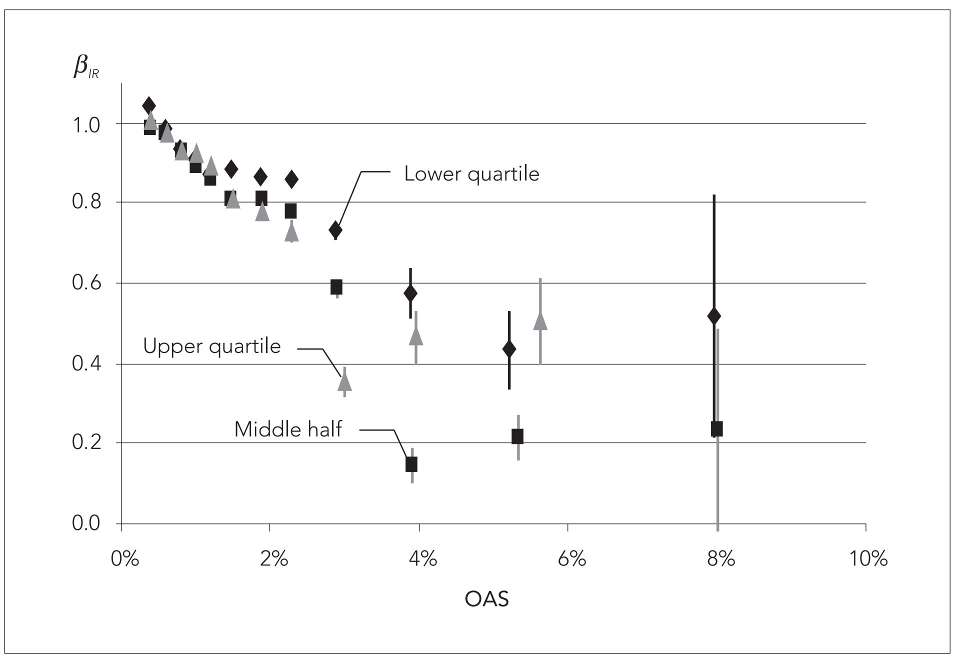


Figure 7

β_{IR} estimates from regressions binned by OAS and forecasted equity volatility



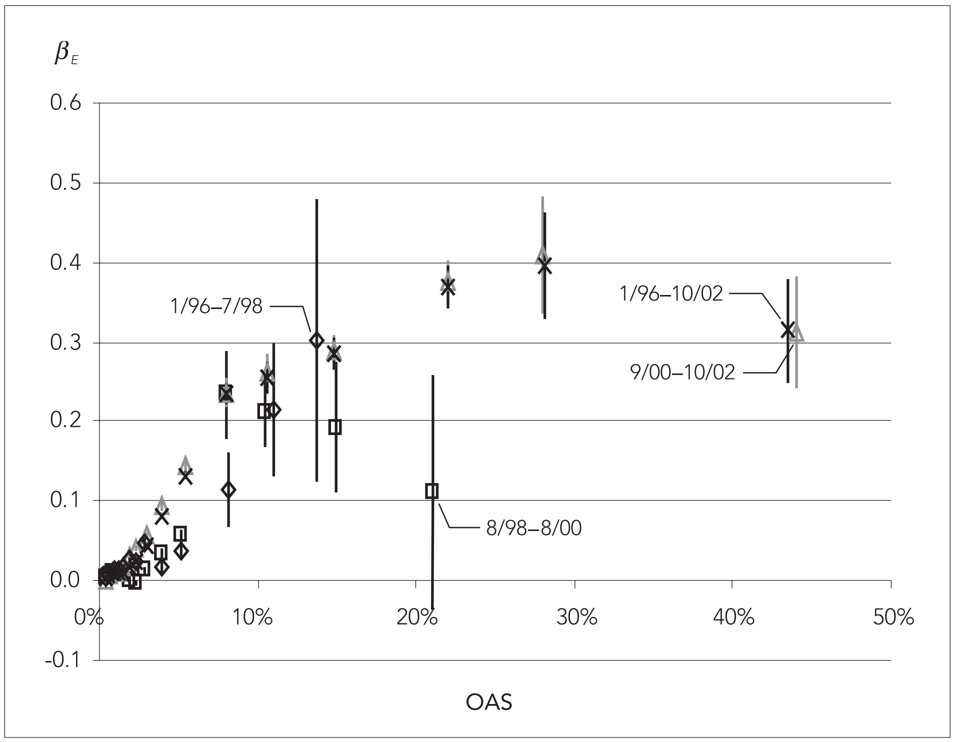
Sample Period

The third test is of dependence of β_E and β_{IR} on sample period. It would be surprising if the relationship were absolutely fixed. But, given known inputs, a reasonable level of stability is essential to the plan of using the empirical relationship for risk forecasting. Based on these 6+ years of data, covering the stock market bubble (1996-2000) and subsequent bust (2000-2003), the Asian currency crisis (1997), the Russian debt default, LTCM collapse and credit crash (1998) and the US economic cycle from rapid growth to recession, the empirical relationship between bond, interest rate and equity returns appears fairly stable.

Figures 8 and 9 and Table 4 in the Appendix show the results of an analysis grouping the data into three periods and by OAS. The three periods are 1/1996–7/1998 (ending immediately before the 1998 credit crash), 8/1998–8/2000 (taking in the credit crash and peak in stock market bubble), and 9/2000–10/2002 (post-bubble bear market).

Figure 8

β_E estimates from regressions grouped by sample period and OAS

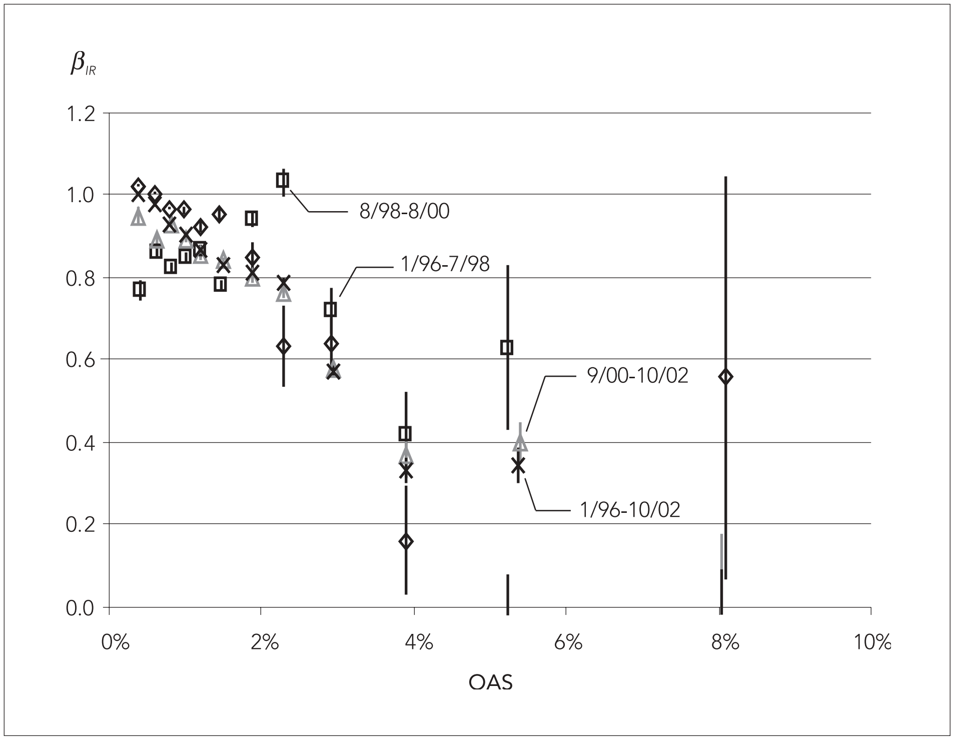


In the earliest sample period, we have relatively little data at high OAS, and the statistical uncertainty on the parameter estimates is large. In both the earliest and the most recent periods, the estimated values of β_E are very similar at low OAS, below 350 bp. The higher OAS bins for the earliest period deviate from the trend, jumping down to near 0 before rising back up again. In the middle sample period, β_E lies below the estimates for the earlier and later periods for OAS < 350 bp, but rises at approximately the same rate with OAS as in the other two periods at OAS > 250 bp. Broadly, in all three sample periods, β_E remains low up to OAS ~ 350 bp, then rises fairly rapidly with increasing OAS, leveling off at around 0.3 for OAS > 1000 bp.

We see no apparent pattern in the behavior of β_{IR} for the different sample periods. In the middle period, which includes the Russian default and LTCM collapse, the lower OAS

Figure 9

β_{IR} estimates from regressions grouped by sample period and OAS



ranges behave somewhat anomalously, but the error bars are comparatively large. What connection, if any, might exist between these observations escapes us.

Equity Return Sign

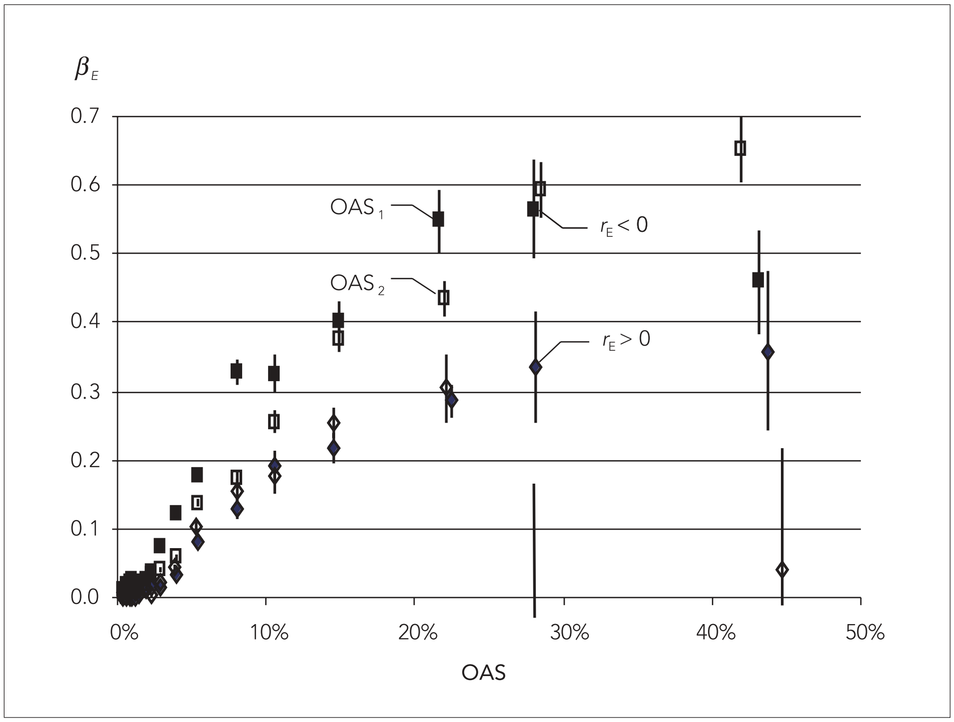
A number of studies have found evidence of increased correlation between aggregate returns to various asset classes in down markets relative to up markets. Here we find as well a substantial difference in linkage between bond and equity returns for events with positive or negative equity return.

Figure 10 shows the estimates for β_E and β_{IR} for positive and negative equity excess return events. There is a very clear, systematic and substantial difference—by more than a factor of five in some OAS bins—between the estimates of β_E for the two subsamples, with negative return events having the larger β_E .The difference in bond-equity return *correlations* between the positive and negative return groups is not quite as dramatic as that between the β_E's, as the square root of the semivariance of bond returns for the negative return events is about 30% greater than for the positive return events, while the corresponding values for the equity returns differ by less than 2%. Some of the difference in β_E's is therefore attributable to asymmetry in the bond returns, with a longer tail on the downside.

Comparing Figure 10 with Figure 8 makes clear that this is not just a misidentified time dependent relationship—that is, a result of there being more positive returns prior to 2000 and more negative returns after, together with the bond-equity linkage becoming stronger since 2000. The slope and highest levels reached by β_E are about 50% greater for the negative return data in Figure 10 than for the largest period dependent estimates shown in Figure 8.

Figure 10

β_E for the full data set, grouped by positive and negative equity excess return and bond OAS. There were approximately 92,000 positive return events and 84,000 negative return events in the sample. The filled data points, labeled OAS_1, are derived by grouping on OAS at the start of each return period (as is done for all the other analysis in this paper). The unfilled points, labeled OAS_2, are derived by grouping on OAS at the end of each return period.



The effect is largest in the most recent sample period (9/2000–10/2002), weaker in the middle period (9/1998–8/2000) and not visible in the earliest period (1/1996–8/1998).

What could account for this anomalous finding? One possibility is simply the non-linearity of the relation between bond and equity returns: a positive equity return should have a smaller impact on bond return than a negative equity return of the same magnitude, because—in any plausible model—bonds with smaller spreads are less sensitive to equity return than bonds with larger spreads, all else equal. A positive equity return corresponds to bonds moving to smaller spreads, and therefore averaging over a range of lower equity exposures, while a negative equity return corresponds to bonds moving to larger spreads, averaging over higher equity exposures. We therefore expect some asymmetry of the measured equity exposures in the positive and negative equity return groups.

In this model, the size of the asymmetry would depend on how rapidly a bond's equity exposure varies with OAS, and on the magnitude of the equity returns, which determines the degree to which the measurement "samples" the non-linearity of the return relationship. However, we do not need to precisely quantify these effects in order to determine whether they are sufficiently large to explain the observations. A simple way to assess the size of the effect is to group bonds in the regression by final OAS in each return period rather than by initial OAS. A bond with given ending OAS would, on average, have had a *higher* initial OAS in the positive equity return data, and therefore had a *greater* average sensitivity to the equity return than a bond with the same ending OAS in the negative equity return data, which would on average arrive from a *lower* initial OAS and therefore have *smaller* average equity exposure. This is exactly the opposite asymmetry to that expected when grouping by initial OAS.

Figure 10 shows the results of binned regressions grouped both ways. If the exposure asymmetry were largely the result of the variation of β_E with OAS, we would expect the unfilled data points (grouped by ending OAS) to show the opposite asymmetry from the filled data points (grouped by starting OAS). What the figure shows is that, although the degree of asymmetry is smaller for the unfilled data points than for the filled, it is still substantial. We conclude that although there is some asymmetry due to nonlinearity of the bond-equity return relationship, it is not large enough to account for the observations. We therefore seek another explanation.

A possible alternative arises from the "agency problem" associated with bonds.[11] For given firm value, the equity holders, and therefore presumably the firm's managers, have an interest in minimizing the value of the outstanding debt, thereby maximizing the value of the equity. A variety of managerial actions can effect changes in this direction. In a Merton model framework, for example, actions whose effect is to increase the expected volatility of the firm value will increase the equity value with an offsetting decrease in bond value. Similarly, increasing leverage by repurchasing shares or issuing new debt will increase equity value at bondholders' expense. More generally, there are a variety of actions that may be taken by a firm's managers to increase shareholder value at bond-holders' expense. Given the incentives, presumably all such actions will be designed to produce positive firm-specific equity returns, leading to the observed reduction in β_E for positive equity excess return events relative to negative. That is, the positive equity return events will consist partly of events with positive impact on overall firm value, tending to increase the value of both bonds and equity, and partly of events with no net effect on firm value, where the positive equity return is compensated by a negative bond return. (In practice, most events will include both firm value changing and value transfer effects jointly.)

This hypothesis suggests testing an alternative specification in place of Equation 1. The events that would cause opposite sign changes in equity and bond value are firm specific, resulting from management actions. Therefore, if instead of relating a bond's return to the overall equity return, we expose it separately to the common-factor and firm-specific components of return, we would expect to see the β_E difference confined to the latter.

The alternative model is:

Equation 2
$$r^t_{B_{ISS}} = \beta_{IR} r^t_{B_{GOV}} + \beta^{common}_E r^t_{E_{ISS-common}} + \beta^{specific}_E r^t_{E_{ISS-specific}} + \varepsilon^t_B .$$

The common-factor return for each stock, $r^t_{E_{ISS-common}}$, is obtained from the USE3 model.

[11] We are indebted to a participant in the NYU mathematical Finance Seminar for this suggestion.

The firm-specific return, $r^t_{EISS-specific}$, is the residual after subtracting the common-factor from the excess return.

The results of estimating this model on binned data, grouped by positive and negative equity excess return, are shown in Figure 11 and in the Appendix in Table 6. There is a fairly clear indication in the results that the β_E's for the common-factor and specific returns of the negative return data and the common-factor returns of the positive return data are similar, while the β_E's for the specific returns of the positive return data are systematically lower than the other three, generally by fairly large factors. These results support the idea that management actions are responsible for the reduced dependence of bond returns on positive equity returns relative to negative.

This idea has been examined recently by Alexander, Edwards and Ferri (2000), and by Maxwell and Stephens (2002). Alexander, Edwards and Ferri use reported daily trade prices (from the Nasdaq FIPS system, since superseded by TRACE) for a small set of high-yield bonds with corresponding daily equity returns. They segregated the returns surrounding public announcements of corporate events, such as new debt or equity issuance and events affecting the expected volatility of the firm value, which might be expected to have differing impacts on equity and bondholders. They find a positive correlation between bond and equity returns overall. However, from the returns surrounding the news announcements, they find (weak) evidence of negative correlation of bond and equity specific returns.

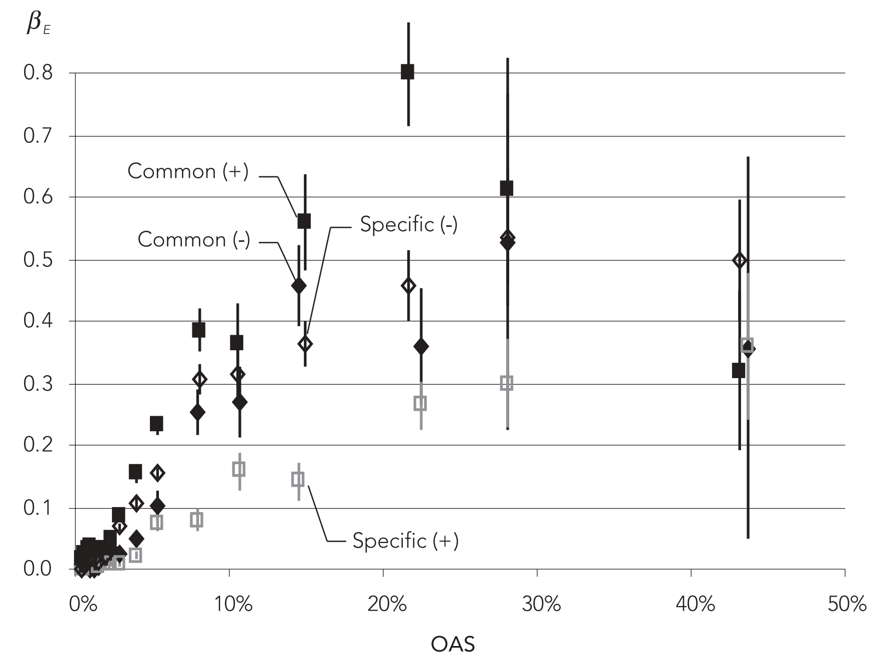
Maxwell and Stephens use monthly bond prices from Lehman Brothers spanning the period 1973 to 1997, together with corresponding equity returns to look for a wealth transfer associated with share repurchases, which, by increasing firm leverage, increase the likelihood of default. They find significant evidence of negative bond specific returns together with positive equity specific returns contemporaneous with announcement of a share repurchase. Moreover, they find that the magnitude of the returns increases with increasing repurchase size, consistent with the expectation of the wealth transfer hypothesis. Finally, the size of the effect is very substantially greater for firms rated below investment grade than for investment grade issuers.[12]

The findings of these papers are consistent with the observations presented here, and support the idea that the smaller β_E associated with positive specific returns is due to management initiated wealth transfers from bondholders to shareholders in a subset of the return events.

[12] Both studies examined anomalous returns defined in different ways and differently from our specification. Alexander, Edwards and Ferri examine "excess" return, defined similarly to our specific return, except that their common factor return is, effectively, just a pair of market index returns, one stock, one bond—with all stocks having unit exposure to the first and all bonds unit exposure to the second—rather than arising from a multiplicity of common factors. Maxwell and Stephens test "abnormal" returns. For stocks this has the same meaning as that of Alexander, Edwards and Ferri, using the CRSP dataset to define the market index. For bonds, they subtract the return to a matched Treasury bond (in our case, equivalent to assuming $\beta_{IR} = 1$), and also a trailing moving average excess return.

Figure 11

Exposures of bonds to common and firm-specific equity returns, grouped by sign of equity excess return and by OAS.



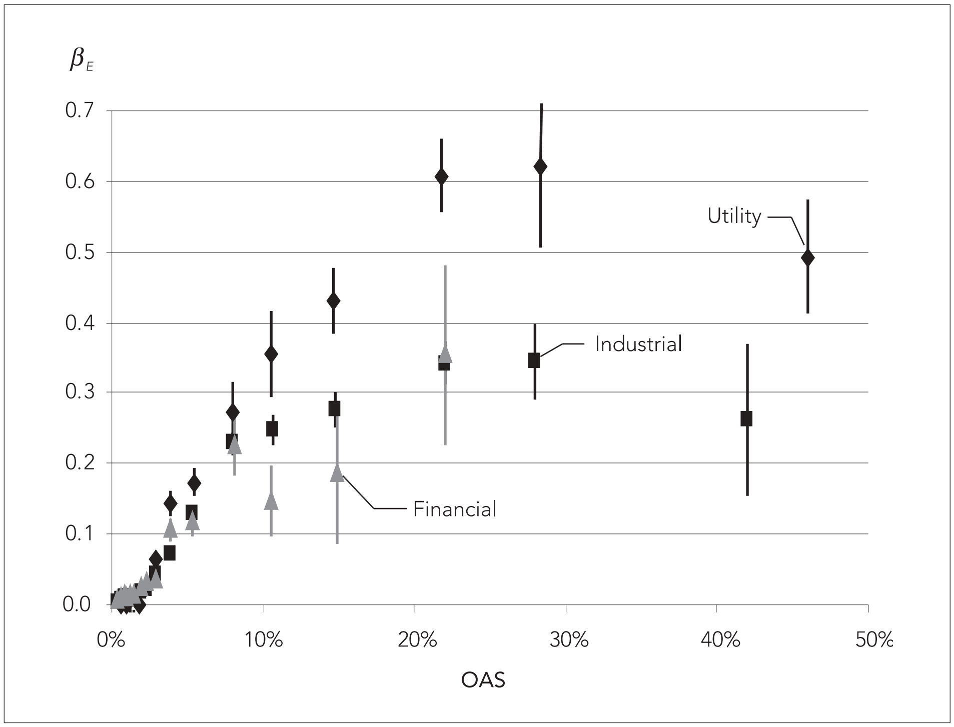
Sector

Financial firms pose a particular challenge for structural credit risk models, due to their high degree of leverage and partial hedging of assets and liabilities. Banks, for example, may have 12:1 or greater ratios of debt to equity. The US federal housing enterprises, Fannie Mae and Freddie Mac, have ratios as high as 40:1, but are generally considered very safe credits (in part, of course, because of their implicit government guarantee). And many firms usually thought of as "industrial" have captive financial subsidiaries, with corresponding leverage. A naïve structural calculation might indicate that these firms are very close to default, even though the debt of such firms generally trades with low spreads and carries strong agency rating. (Otherwise they could not finance their operations effectively in the short term money markets.)

The empirical approach described here has no manifest difficulty in modeling the public debt of financial firms—we assume that it can be treated just like bonds of other types of firms, with the OAS as a measure of market perception of creditworthiness. To test this assumption, we grouped the estimation universe by market sector, looking separately at bonds issued by firms categorized as financial, industrial and utility. The results are displayed in Figure 12. Up to about 400 bp OAS, all three sectors have virtually indistinguishable dependence of β_E on OAS. At higher OAS, the utility bonds appear to have systematically larger values of β_E than either the industrials or financials, though not by significantly more than the statistical parameter standard deviation. The industrial and financial bonds give similar estimates of β_E. From this result, it seems reasonable to use a common measure β_E, independent of sector, though one might plausibly argue for treating the utility bonds differently from the others at the highest OAS levels.

Figure 12

β_E estimates from data grouped by sector and OAS. Data are in the Appendix in Table 7.



Fallen Angels

There is no obvious reason to expect bonds of so-called fallen angels—issuers having an investment grade rating when they issue bonds subsequently rated below investment grade—to behave differently from other low-grade bonds of similar current credit quality. Certainly, in a structural framework, the past agency rating has no bearing at all on current default probability estimates.[13] At the same time, bonds of fallen angels may have different characteristics than speculative-at-issue high-yield debt. A recent report from Standard & Poor's observes (Vazza and Cantor, 2003)

> "Covenant protection measures of these formerly investment grade issuers are inferior to the standard covenants of original issuer high yield. Buyers look to those issuers that have limitations on debt incurrence or restricted payments, tests and strong negative pledge clauses, not typically found in investment grade deals."

There are also arguments suggesting that, due to seasoning or momentum effects, fallen angels may have greater default rates than new issues of equivalent agency rating, at least over the short term (Brady, 2003). To the extent that market participants are aware of these influences and their impact, one would expect these differences in credit risk to be reflected in the market price, or OAS, of the bonds.

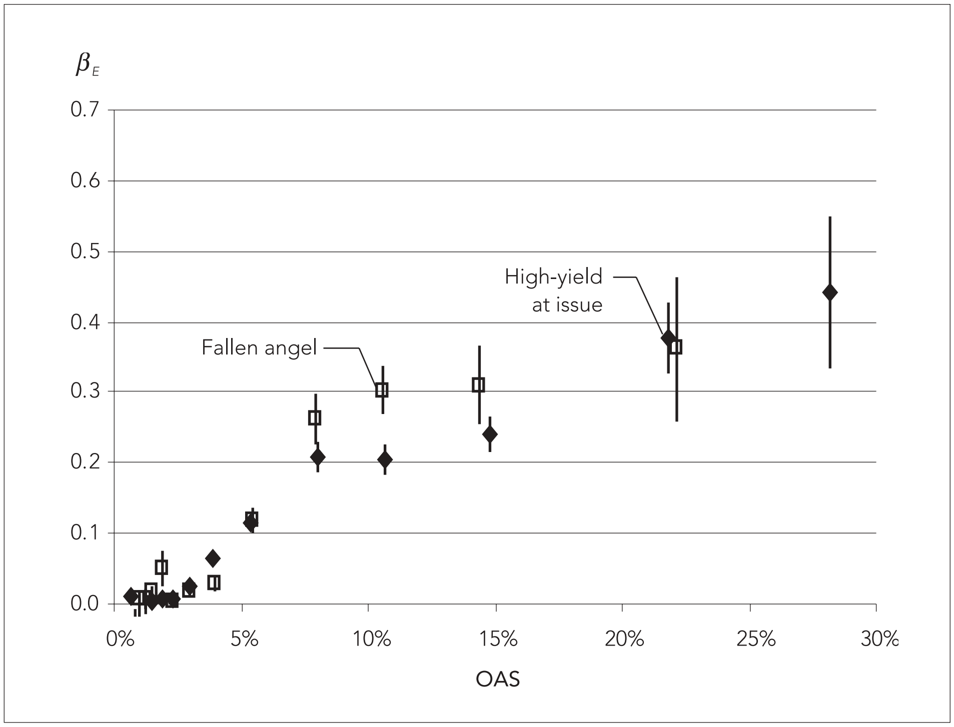
Figure 13 shows estimates of β_E from high yield bonds (carrying "speculative" ratings, below BBB-/Baa3 on the return date) grouped into "natural" high-yield bonds (speculative

[13] Although one recent model does incorporate dependence on the history of a firm's value. See Giesecke, K., 2002.

at issue) compared to values estimated from fallen angels (rated BBB-/Baa3 or better at issue). Because we are including only the comparatively small number of low-rated bonds in this analysis, the statistics on the estimates are considerably poorer than in the other regressions, especially at low OAS. The results show no indication of any difference in the equity exposure of fallen angels as compared to natural high-yield bonds, at least when grouped by OAS.

Figure 13

Estimated β_E for fallen angels and bonds rated below investment grade at issue. Fallen angels are bonds rated BBB-/Baa3 or better at issue, and rated BB+/Ba1 or worse at the start of the month for which the return is calculated. Data are in the Appendix in Table 8.



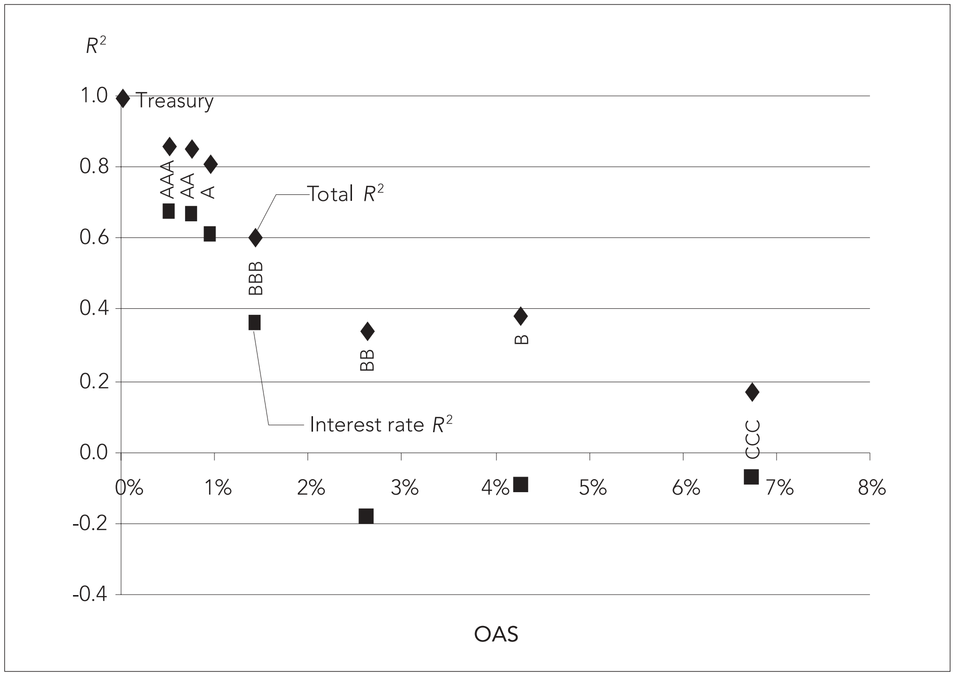
5. Application to Risk Modeling

Traditional models of risk for corporate bonds are based on the attribution of bond returns to interest rate and residual credit spread factors. Interest rate factor returns, derived from either returns to government bonds or changes in swap rates, are exogenous to the corporate bond universe. Spread factors are derived by categorizing credit-risky bonds as, e.g., AA-rated industrials, A-rated financials, and so on, then regressing a set of common spreads on bond returns residual to interest rates. The spread factors are often referred to as credit risk factors, although they are not directly due to credit risk in the sense of issuer-level risk due to downgrade or default. Rather, they account for market-wide repricing of credit risk for groups of similar issuers. Historical timeseries of these factor innovations (“returns”) are then used to construct forecasts of the future distribution of returns.

This attribution works quite well for high-grade corporate bonds, for which interest rate changes are the dominant source of cross-sectional return variation, with common spread factors accounting for a quarter to half of the remaining return variance (see Figure 14).

Figure 14

R^2 of the Barra fixed income risk model for US corporate and Treasury bonds grouped by rating, based on monthly returns from 1/2000 through 10/2002[14]. The model has the form $r^t_{BISS} = r^t_{BGOV} + (-D^S_B)\Delta s^t_{CISS} + \varepsilon$, relating the return for a corporate bond to the return on a government bond with the same interest rate exposures and the change in the common spread Δs^t_{CISS} to which the bond is mapped. (The spread exposure is minus the bond's spread duration[15].) The OAS for each point is the average spread, adjusted for embedded options, of the bonds in that rating group. The points labeled "Interest rate R^2" show the fraction of return variance attributable to government bond returns (or default-free interest rates) alone. The points labeled "Total R^2" show the corresponding result after estimating the spread common-factor returns. The negative "Interest rate R^2" for bonds rated BB and below is due to the negative correlation of high yield and government bond returns over the sample period.



However, as we move down the credit quality spectrum, interest rates and common spreads account for a rapidly decreasing share of return. Changes in government bond yields or swap rates and common spread factors tell us very little about returns to bonds rated BB and below. After incorporating spread factors, the R^2 of the model ranges from below 40% for bonds rated BB and B to below 20% for bonds rated CCC.

In order to obtain a model for application to risk forecasting, we will have to estimate parameters of curves such as those of Figure 1. These are plots of functions of the form

Equation 3a
$$\beta_{IR} = 1 - \alpha_1 (1 - exp(-\alpha_2 \cdot OAS))^{\alpha_3}$$

Equation 3b
$$\beta_E = \alpha_4 (1 - exp(-\alpha_5 \cdot OAS))^{\alpha_6}$$

where $\alpha_1 \ldots \alpha_6$ are parameters defining the shapes. The choice of these functional forms is somewhat arbitrary, dictated only by the desire that the functions be asymptotically constant at large OAS, lie between 0 and 1, and be described by a small number of parameters, to avoid overfitting. We require fitted functional forms such as these, rather than simply using the results of, say, binned regressions, for two reasons. One is that we expect the true relationship between bond and equity returns to vary smoothly and monotonically with credit quality, whereas binned regressions would give us discontinuous and possibly non-monotonic behavior (due to statistical errors). Secondly, at extreme values (low or high) of the OAS, we have relatively little data for estimation of β_{IR} and β_E , so the

[14] The model is described in detail in "The Barra U.S. Fixed Income Risk Model," which can either be downloaded from http://www.barra.com/support/library/us_fixed_income_model.pdf or requested from your Barra representative.

[15] Spread duration is the negative fractional price sensitivity of a bond to change in OAS. For a fixed-rate bond, standard calculation methods equate this to the effective (or option adjusted) duration. For other types of securities, such as floating-rate notes, the two quantities may be very different.

estimates will be quite noisy. By imposing functional forms with reasonable limiting behavior, we can obtain usable estimates for the values in these extremes.

We are also free to adapt the heuristic functional forms of β_{IR} and β_E and to accommodate dependence on additional characteristics of bonds or firms. For example, we found that β_E depends on duration as well as OAS, and therefore in application we modify Equation 3b to

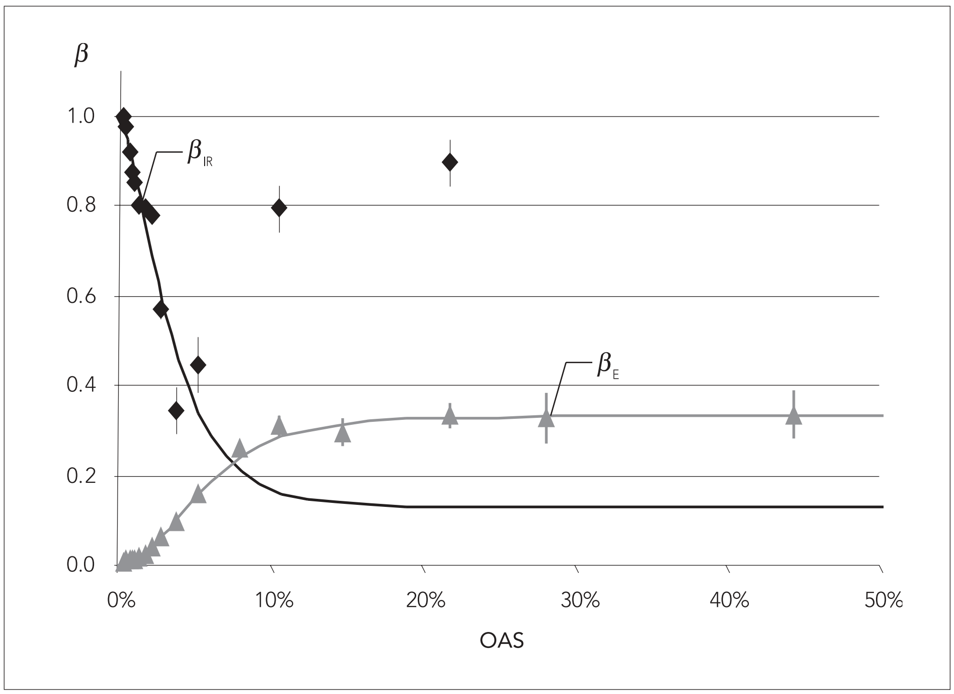
Equation 3b′

$$\beta_E = \alpha_4(1 - exp(-(\alpha_5 + \alpha_7 D) \cdot OAS))^{\alpha_6}$$ where D is the bond's duration.

Figure 15 shows the functions β_{IR} and β_E of equations (3a, b) with parameters estimated by GLS from the same data as the binned regressions. The figure also shows the binned regression parameters for comparison, as in Figure 3. These simple functional forms manifestly provide reasonable smooth interpolation and extrapolation of the empirical bond return relationships.

Figure 15

General least squares fit of the parameterized functions (3a, b) with the same data as used for Figure 3. The binned regression estimates are shown for comparison. The parameters are *not* estimated from the binned regression results.



A simple application of the heuristic representation for β_E is the forecasting of the distribution of bond prices or spreads. Holding all but the equity contribution fixed,[16] for an individual bond, Equation 1 may be integrated to give

Equation 4

$$\int_{B_i}^{B_f} \frac{dB}{B\beta_E(s(B))} = ln(1 + r_E)$$

where B_i and B_f are the starting and ending bond prices and $s(B)$ is the OAS as a function of bond price. With the further approximation that the spread duration, $D_B^S = \left(-\frac{1}{B}\frac{\partial B}{\partial s}\right)$, is

[16] Generically, Equation 1 describes a path-dependent relationship between equity and interest rate changes and the bond return, because the β functions are not components of an exact differential. This complication goes away if we restrict attention to bond returns implied by equity returns alone.

approximately independent of bond price, we obtain the simplification approximately independent of bond price, we obtain the simplification

Equation 5
$$\int_{s_i}^{s_f} \frac{ds}{\beta_E(s)} = -\frac{1}{D_B^S} ln(1+r_E),$$

where s_i and s_f are the initial and final bond spreads.

Given a model for the distribution of the right hand side of this equation (for example, normal), we obtain a distribution for the bond spread. Some examples are shown in Figure 16, based on volatility forecasts and initial OAS's for bonds as of June 2002. The variation in width of the densities arises both from differences in initial values (smaller initial spreads imply smaller initial β_E), but also from substantial differences in the equity volatility forecast. Broadwing (Cincinnati Bell) and Crown Holdings both have forecasted annualized equity volatility of about 80%, but in June 2002, a Broadwing bond had an OAS of about 400 bp, and therefore a smaller β_E than Crown Holdings debt, which had a spread of almost 2000 bp. On the other hand, Lyondell Chemical, with a June 2002 OAS of 970 bp, and forecast equity volatility of just 37%, has a much narrower probability density for the spread a year later than that of Metris, with lower initial OAS of 890 bp (therefore lower initial β_E), but higher forecast volatility of 66%.

For a portfolio manager, the density curves of Figure 16 clearly leave out important information, namely the correlations in the outcomes. The issuers of these bonds include an airline, a chemical producer, a consumer credit firm, two telecoms and a packaging manufacturer. We expect that the positive and negative outcomes in the figure are neither perfectly correlated nor completely independent. One approach to predicting these correlations is through a factor model of returns.

The factor-based approach to risk modeling mentioned earlier for bonds was first developed by Barra in the 1970s, for application to equities, and widely adopted since then. The model decomposes asset returns into components attributable to common factors and residuals (specific returns), and constructs forecasts for the factor return distributions, or at least their second moments (variances and covariances). Such models are extremely useful for portfolio risk forecasting, as they permit the aggregation of risk forecasts for individual assets to the portfolio.[17]

As already noted, the standard "interest rates plus spreads" factor model for bonds works quite well for higher quality investment grade bonds, but not very well for low-grade bonds, primarily because changes in default-free interest rates are not useful explanatory factors for such bonds, and sector/rating spreads alone don't contribute a great deal to explaining their returns.

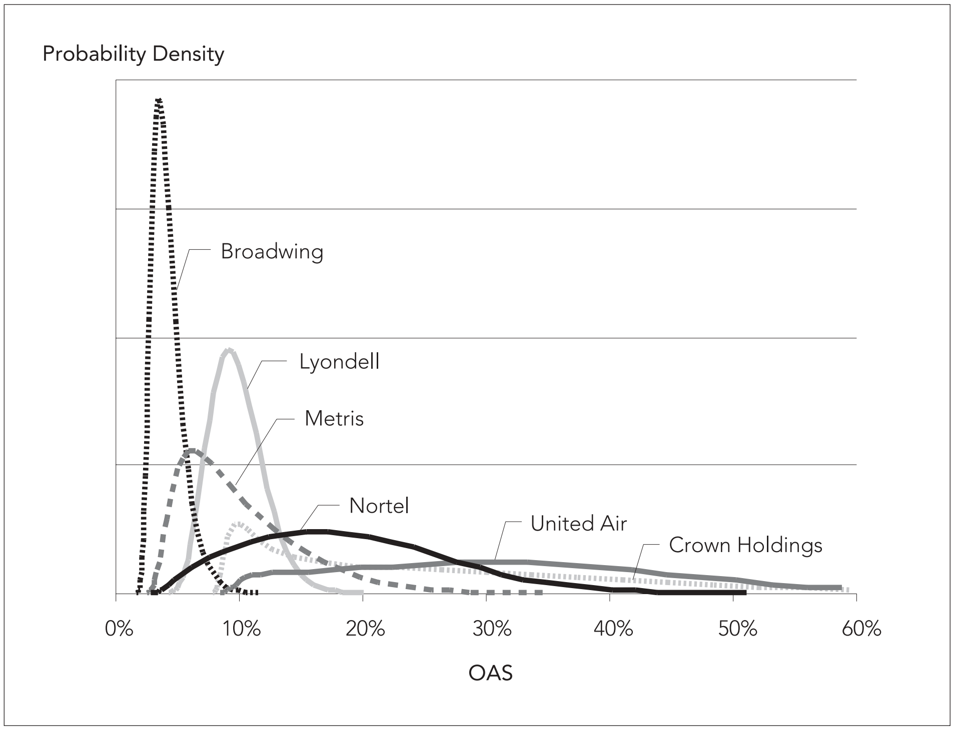
On the other hand, as we've seen, returns to low-grade bonds are explained quite well

[17] The common factor and specific return distributions may be estimated from historical asset return data, from derivatives markets (e.g., option implied volatility), or by some combination. Asset exposures may be specified in the model definition (e.g., based on the issuer's sector), numerically derived (as in the calculation of interest rate exposure using a bond valuation model for high-grade bonds), or statistically estimated (e.g., by style analysis, for mutual funds).

by equity returns. And, as noted in footnote 10, the volatility forecasts of Barra's US equity model (USE3) are quite accurate overall. Combining an interest rate factor model with an equity factor model, and using Equations 1 and 3a and 3b or b'), we obtain a model of the common factors and exposures to which bonds are exposed across the spectrum of credit quality. That is, a bond with OAS *s* is exposed to interest rate factors with exposures given by it $\beta_{IR}(s)$ multiplied by the computed interest rate factor exposures $(-D_{B,i})$, and to equity factors with exposures $\beta_E(s)$ multiplied by the factor exposures of the equity.

Figure 16

Spread probability distributions for bonds of various issuers at a one-year horizon, assuming a lognormal distribution of equity returns, using the best-fit parameter estimates for β_E and spread and equity volatility forecasts as of June 2002.



As shown in Figure 3, the explanatory power of this model falls to a low level of $R_2 \sim 0.14$ at boundary between investment and high-yield bonds. Motivated by the earlier study by Collin-Dufresne, *et al.*, and by our observations that the sector/rating spread factors contribute roughly 0.1 to 0.2 of improvement in R^2 for intermediate to lower-grade bonds, we extend the model by looking for additional factors to explain the residuals from Equation 1.

The simplest model (though perhaps having too many factors) is a sector/rating spread model of the residuals. The residuals of Equation 1 are regressed on a spread model with each bond exposed to one spread factor, based on sector and rating, with magnitude equal to minus the spread duration. This is very similar to the method used to estimate the "interest rates plus spreads" model currently delivered by Barra, except that rather than fitting the spread model to the residuals after accounting for interest rates alone (with unscaled exposures), the spread model is now applied to the residuals after accounting both for interest rates (with β-scaled exposures) and equity returns. The resulting model of returns can be written as

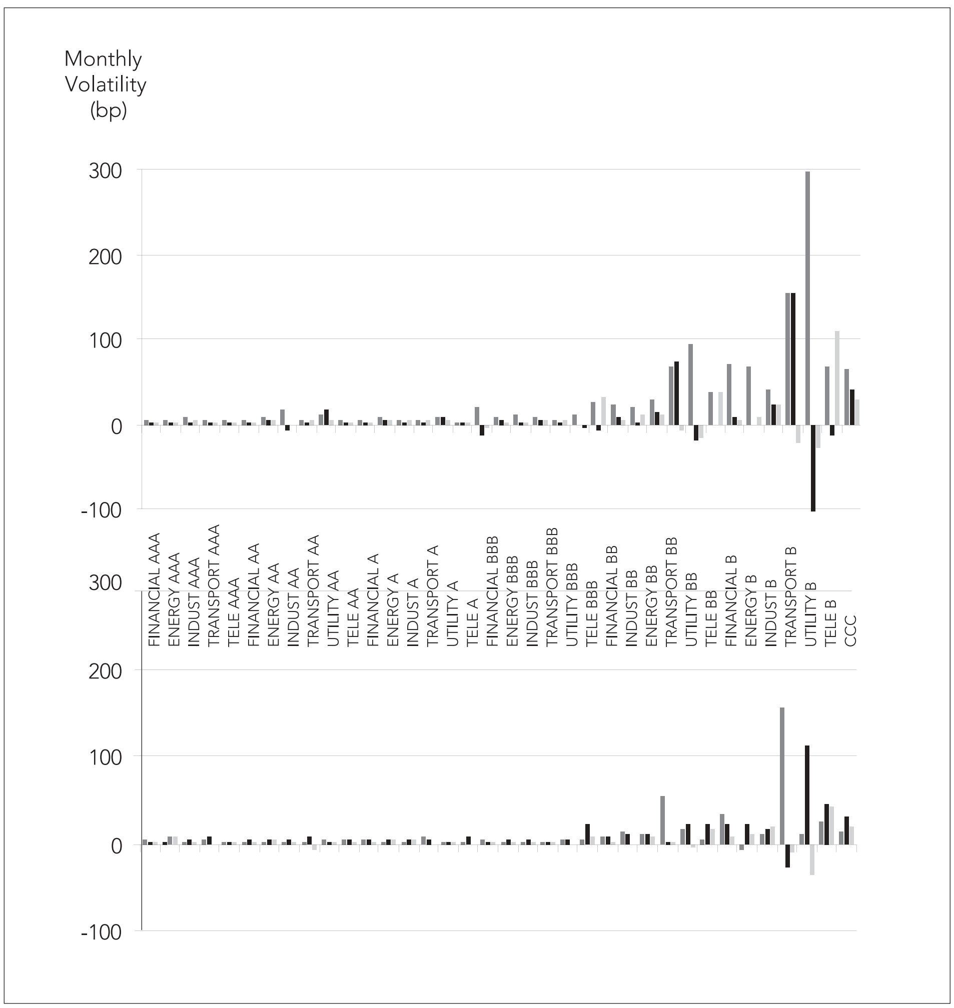
$$r^t_{BISS} = \beta_{IR} r^t_{BGOV} + \beta_E r^t_{EISS} + (-D^S_B)\Delta s^t_{CISS} + \eta^t_B,$$

where Δs^t_{CISS} is the "spread" change for the issuer's sector C_{ISS} and η^t_B is the remaining residual return. Note that the spread changes Δs^t_{CISS} are not changes to market spreads for bonds in each sector: first because a portion of each bond's spread change has already been accounted for by the equity return, and second because the interest rate exposures are scaled by $\beta_{IR} < 1$. We refer to them as spread changes because the bonds' exposures to them are equal to their spread durations, and because for high-grade bonds, for which $\beta_{IR} \sim 1$ and $\beta_E << 1$, they are very close to the sector/rating spread change.

The added explanatory R^2 produced by this extension is shown in Figure 17, comparing the R^2 of the current Barra model, a model of bond returns based on interest rates only, the model of Equation 1, and the model of Equation 6. The calculations are based on the regression formulas using the heuristic functional forms of Equations (3a) and (3b'). In each case, bonds have been grouped by coarse agency rating for the R^2 calculation.

Figure 17

R^2 for bonds grouped by coarse agency rating (average of S&P and Moody's), for four models of bond returns, as described in the text. The sample period is 1/2000 to 10/2002. The curves labeled "Equation 1" and "Equation 6" refer to the return attribution formulas in the text. The other two curves refer to attributions to government bond returns ("rates") with unit exposure $(\beta = 1)$, with and without spreads. The "rates and spreads" model is Barra's current US fixed-income model for corporate bonds, and works nearly as well as the model of Equation 6 for bonds rated BBB and above.

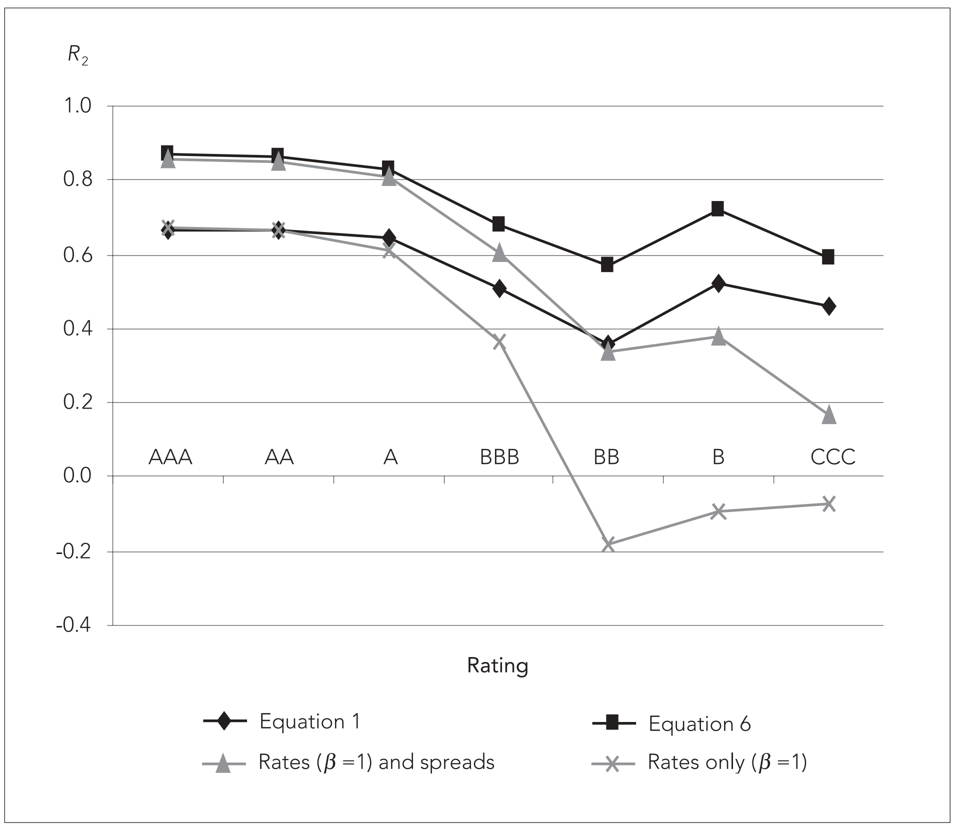


The sharp dip of Figure 3 at around 400 basis points is now visible as a dip in the curve labeled "Equation 1" for BB rated bonds. The rating categories include bonds with a wide range of OAS's, thereby smearing the details of the earlier figure.

Further insight into the difference between the rates-plus-spreads model and the one based on Equation 6 is evident from Figure 18, which shows the first three eigenvectors of the spread covariance matrix from both models. The eigenvectors have been scaled by the square root of the corresponding eigenvalues (their volatilities) to show the contribution of each factor to residual spread volatility. With the single exception of the transport-B spread, the scaled eigenvectors of ECR residual spreads are substantially smaller than those of the rates-plus-spreads model.[18] This is just a consequence of the fact that the equity return accounts for an appreciable component of return that is instead accounted for by spreads in the rates-plus-spreads framework.

Figure 18

First three eigenvectors scaled by their volatilities for the spread covariance matrices of the rates + spreads model (upper figure) and from Equation 6 (lower figure). With the exception of B-rated transports, the magnitudes are appreciably smaller in the lower figure than in the upper. The large correlation between B-rated transports and utilities evident in the upper figure also disappears in the lower. The persistent large transport-B spread factor volatility is attributable to the 9/11/01 event.



6. Summary

This paper describes an empirical attribution of corporate bond returns to returns of corresponding default-free bonds (or, equivalently, interest rates) and the issuer's equity. We demonstrate, first, that a bond's option adjusted spread (OAS) can be used as a measure of its exposure to default-free bond and equity returns. A corporate bond's exposure to interest rates decreases with increasing OAS, while the exposure to the issuer's equity increases. Very high quality corporate bond returns are explained almost entirely by interest rate changes, which account for upwards of 80% of return variance—cross-sectionally and over time – while equity returns account for 40% or more of the return variance of low grade bonds, those with OAS's above 10%. Interestingly, returns to bonds of intermediate credit quality do not appear to be significantly explained by either exogenous source. Although we cannot explain the underlying drivers of return for these bonds, we nevertheless can identify common factors similar to sector/rating spreads, which raise the overall R^2 of the model to above 50% for all levels of credit quality.

We studied several dimensions of variation in bond and equity attributes to find additional factors affecting β_{IR} and β_E. We did not find an economically significant effect for most of the factors, but two did have an impact. Although the effect is not large, the equity exposure of higher duration bonds increases more rapidly with OAS than lower duration bonds. We are able to account for this, at least approximately, by a small adjustment to the heuristic functional form used to represent β_E.

An intriguing finding is the substantial difference in β_E between positive and negative equity excess return events. On further analysis, this difference appears to be confined to positive equity specific returns, which are much more weakly linked to bond returns than are negative specific returns or common factor returns of either sign. A possible explanation attributes the phenomenon to managerial actions that shift firm value from bondholders to shareholders. Such actions will always be intended to produce positive equity specific return. Whether this is the full explanation for the observations remains an open question.

Equity factor-based risk models are constructed from equity return data, which fairly directly measures changes in fundamental firm valuation factors. A return model exposing bonds to equity factors therefore affords more insight into the sources of risk and return than one based on attribution of bond returns to spreads. The ECR model, explaining bond returns in terms of interest rate, equity and spread factors accounts for a comparatively large fraction of return variance for bonds of all levels of credit quality. We expect it to provide a significant improvement in the quality of risk forecasts over more traditional bond risk models.

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Tables

Table 1

Aggregate regression results for Equation 1 on binned data, for the period 1/1996 to 10/2002. The mean OAS for each bin is an unweighted average for all bonds in the bin. *t*-statistics for β_{IR} and β_E are shown in the adjacent columns. (Data for Figure 3.)

OAS Range	Mean OAS	Number of Events	R^2	β_{IR}	t	β_E	t
0-50 bp	39 bp	19,265	0.91	1.00	220.6	0.01	12.6
50-70	60	24,189	0.90	0.97	282.5	0.01	23.9
70-90	80	22,858	0.82	0.92	198.9	0.01	17.4
90-110	100	19,025	0.71	0.88	116.6	0.01	14.2
110-130	120	16,878	0.64	0.85	109.4	0.01	11.1
130-170	149	26,397	0.53	0.80	111.0	0.01	15.0
170-210	188	16,387	0.46	0.79	74.3	0.02	15.2
210-250	228	9,738	0.33	0.78	50.7	0.04	12.3
250-350	293	11,842	0.15	0.57	26.0	0.06	18.1
350-440	391	5,725	0.14	0.34	7.0	0.10	12.7
440-700	538	5,448	0.22	0.45	7.3	0.16	18.9
700-950	806	1,458	0.37	-0.21	-0.9	0.26	12.8
950-1200	1059	688	0.41	0.80	2.3	0.31	14.0
1200-1900	1481	777	0.39	-0.23	-0.5	0.29	9.5
1900-2600	2190	260	0.46	0.90	1.0	0.33	11.9
2600-3200	2817	88	0.47	2.65	1.3	0.33	5.8
3200-7500	4450	91	0.37	1.94	0.6	0.34	6.3

Table 2

β_E and β_{IR} from data binned by duration and OAS. One standard deviation error estimates are in parentheses. The data sample is as in Table 1. (Data for Figures 4 and 5.)

	D < 3			3 < D < 4.4			4.4 < D < 5.9		
OAS Range	β_{IR}	β_E	*N*	β_{IR}	β_E	*N*	β_{IR}	β_E	*N*
0-50 bp	0.96 (0.01)	0.00 (0.0)	7783	1.00 (0.01)	0.01 (0.00)	4086	1.03 (0.01)	0.01 (0.00)	2946
50-70	0.92 (0.01)	0.00 (5.4)	5577	0.96 (0.01)	0.01 (0.00)	5016	0.98 (0.01)	0.01 (0.00)	5054
70-90	0.87 (0.01)	0.00 (1.7)	5117	0.94 (0.01)	0.01 (0.00)	3841	0.96 (0.01)	0.02 (0.00)	4158
90-110	0.88 (0.01)	0.00 (4.0)	4154	0.90 (0.01)	0.00 (0.00)	3384	0.98 (0.01)	0.01 (0.00)	3206
110-130	0.92 (0.02)	0.00 (5.7)	3321	0.89 (0.01)	0.00 (0.00)	3089	0.94 (0.01)	0.01 (0.00)	2889
130-170	0.94 (0.01)	0.01 (8.7)	3757	0.90 (0.01)	0.01 (0.00)	4759	0.89 (0.01)	0.01 (0.00)	4965
170-210	0.91 (0.03)	0.02 (5.1)	1537	0.87 (0.02)	0.02 (0.00)	2037	0.85 (0.01)	0.02 (0.00)	3171
210-250	0.71 (0.05)	0.02 (5.9)	932	0.75 (0.04)	0.03 (0.00)	1287	0.75 (0.03)	0.02 (0.00)	1892
250-350	0.40 (0.07)	0.02 (5.8)	1509	0.37 (0.04)	0.03 (0.00)	2374	0.53 (0.03)	0.04 (0.00)	2844
350-440	0.10 (0.09)	0.05 (5.6)	896	0.08 (0.04)	0.05 (0.01)	1582	0.29 (0.06)	0.08 (0.01)	1562
440-700	-0.00 (0.13)	0.08 (8.3)	1017	0.03 (0.07)	0.11 (0.01)	1548	0.16 (0.08)	0.14 (0.01)	1572
700-950	-0.42 (0.53)	0.14 (6.4)	302	0.27 (0.23)	0.21 (0.03)	441	0.39 (0.35)	0.33 (0.03)	423
950-1200	-0.85 (0.84)	0.19 (5.0)	160	-0.05 (0.48)	0.26 (0.03)	227	0.34 (0.56)	0.28 (0.04)	189
1200-1900	-2.83 (0.93)	0.16 (5.4)	229	0.55 (0.49)	0.33 (0.03)	347	0.51 (0.85)	0.33 (0.04)	152
1900-2600	-0.52 (1.52)	0.28 (5.8)	88	0.48 (0.95)	0.40 (0.03)	139			
2600-3200	3.48 (5.11)	0.51 (3.7)	44	2.73 (2.48)	0.33 (0.07)	27			
3200-7500	5.25 (3.13)	0.31 (4.6)	79						

continued on following page

Table 2
continued

	5.9 < D < 8.1			D > 8.1		
OAS Range	β_{IR}	β_E	N	β_{IR}	β_E	N
0-50 bp	1.04 (0.01)	0.00 (0.00)	2438	0.98 (0.01)	-0.00 (0.00)	1525
50-70	1.01 (0.00)	0.01 (0.00)	5086	0.95 (0.01)	0.01 (0.00)	2644
70-90	0.97 (0.01)	0.02 (0.00)	4644	0.91 (0.01)	0.02 (0.00)	4317
90-110	0.98 (0.01)	0.01 (0.00)	3584	0.85 (0.01)	0.02 (0.00)	4120
110-130	0.94 (0.01)	0.00 (0.00)	3251	0.81 (0.01)	0.02 (0.00)	3826
130-170	0.91 (0.01)	0.01 (0.00)	5813	0.76 (0.01)	0.02 (0.00)	6370
170-210	0.88 (0.01)	0.02 (0.00)	4327	0.77 (0.01)	0.02 (0.00)	4904
210-250	0.83 (0.02)	0.02 (0.00)	2363	0.78 (0.02)	0.03 (0.00)	2965
250-350	0.67 (0.03)	0.05 (0.01)	2073	0.61 (0.03)	0.07 (0.01)	2789
350-440	0.64 (0.09)	0.13 (0.02)	601	0.40 (0.06)	0.11 (0.01)	964
440-700	0.57 (0.11)	0.17 (0.02)	643	0.82 (0.12)	0.19 (0.02)	512
700-950	-0.75 (0.32)	0.21 (0.03)	192			
950-1200	0.56 (0.74)	0.25 (0.07)	57			

Table 3

β_E and β_{IR} from data binned by forecast volatility and OAS. One standard deviation error estimates are in parentheses. The data sample is as in Table 1.

(Data for Figures 6 and 7.)

	Lower Quartile			Middle Half			Upper Quartile		
OAS Range	β_{IR}	β_E	N	β_{IR}	β_E	N	β_{IR}	β_E	N
0-50 bp	1.04 (0.01)	0.00 (0.00)	4814	0.99 (0.01)	0.01 (0.00)	9308	1.01 (0.01)	0.00 (0.00)	4646
50-70	0.99 (0.01)	0.01 (0.00)	5901	0.97 (0.00)	0.01 (0.00)	11678	0.97 (0.01)	0.01 (0.00)	5796
70-90	0.94 (0.01)	0.01 (0.00)	5602	0.93 (0.01)	0.01 (0.00)	11011	0.93 (0.01)	0.01 (0.00)	5455
90-110	0.91 (0.01)	0.01 (0.00)	4717	0.89 (0.01)	0.01 (0.00)	9061	0.92 (0.01)	0.01 (0.00)	4665
110-130	0.87 (0.01)	0.01 (0.00)	4130	0.86 (0.01)	0.01 (0.00)	8137	0.89 (0.01)	0.01 (0.00)	4095
130-170	0.89 (0.01)	0.00 (0.00)	6489	0.81 (0.01)	0.01 (0.00)	12786	0.81 (0.01)	0.01 (0.00)	6377
170-210	0.86 (0.02)	0.02 (0.00)	4014	0.81 (0.01)	0.02 (0.00)	7969	0.78 (0.02)	0.02 (0.00)	3987
210-250	0.86 (0.02)	0.01 (0.00)	2345	0.78 (0.02)	0.03 (0.00)	4780	0.73 (0.03)	0.02 (0.00)	2308
250-350	0.73 (0.02)	0.03 (0.01)	2886	0.58 (0.02)	0.04 (0.00)	5808	0.35 (0.04)	0.04 (0.00)	2892
350-440	0.58 (0.06)	0.09 (0.01)	1415	0.14 (0.05)	0.07 (0.01)	2820	0.47 (0.07)	0.08 (0.01)	1367
440-700	0.44 (0.10)	0.13 (0.01)	1358	0.22 (0.06)	0.13 (0.01)	2639	0.50 (0.11)	0.13 (0.01)	1293
700-950	0.52 (0.31)	0.23 (0.04)	357	0.23 (0.25)	0.27 (0.02)	668	-0.74 (0.33)	0.21 (0.02)	328
950-1200	0.41 (0.60)	0.23 (0.04)	179	-0.08 (0.40)	0.31 (0.02)	285	0.92 (0.58)	0.19 (0.03)	163
1200-1900	-0.45 (0.61)	0.37 (0.04)	210	1.07 (0.64)	0.34 (0.03)	352	-0.76 (0.74)	0.24 (0.03)	163
1900-2600	1.40 (1.68)	0.47 (0.06)	65	0.75 (1.48)	0.35 (0.04)	117	1.18 (1.22)	0.37 (0.03)	49
2600-3200	-4.00 (4.00)	0.45 (0.14)	35	6.69 (2.89)	0.65 (0.12)	25	-1.95 (3.14)	0.29 (0.05)	12
3200-7500	2.37 (5.08)	0.48 (0.07)	33	0.50 (6.68)	0.28 (0.13)	24	10.22 (7.70)	0.21 (0.11)	22

Table 4

β_E and β_{IR} from data binned by sample period and OAS. One standard deviation error estimates are in parentheses. The data sample is as in Table 1. (Data for Figures 8 and 9.)

	1/96-7/98			8/98-8/00			9/00-10/02		
OAS Range	β_{IR}	β_E	N	β_{IR}	β_E	N	β_{IR}	β_E	N
0-50 bp	1.02 (0.00)	0.00 (0.00)	17278	0.77 (0.03)	0.00 (0.00)	1091	0.95 (0.02)	0.00 (0.00)	409
50-70	1.00 (0.00)	0.01 (0.00)	18637	0.86 (0.01)	0.01 (0.00)	3511	0.89 (0.02)	0.01 (0.00)	1229
70-90	0.97 (0.00)	0.01 (0.00)	11747	0.82 (0.01)	0.01 (0.00)	7120	0.93 (0.01)	0.01 (0.00)	3210
90-110	0.96 (0.01)	0.01 (0.00)	5287	0.85 (0.01)	0.01 (0.00)	8457	0.89 (0.01)	0.01 (0.00)	4704
110-130	0.93 (0.02)	0.01 (0.00)	2550	0.87 (0.01)	0.01 (0.00)	7542	0.85 (0.01)	0.01 (0.00)	6284
130-170	0.95 (0.02)	0.02 (0.00)	1984	0.78 (0.01)	0.01 (0.00)	9846	0.84 (0.01)	0.02 (0.00)	13834
170-210	0.85 (0.04)	0.03 (0.00)	834	0.94 (0.02)	0.00 (0.00)	5120	0.80 (0.01)	0.03 (0.00)	10022
210-250	0.63 (0.10)	0.02 (0.01)	449	1.03 (0.04)	-0.00 (0.00)	2491	0.76 (0.01)	0.04 (0.00)	6499
250-350	0.64 (0.06)	0.05 (0.01)	652	0.72 (0.05)	0.01 (0.00)	2490	0.58 (0.02)	0.06 (0.00)	8447
350-440	0.16 (0.13)	0.02 (0.01)	254	0.42 (0.10)	0.03 (0.01)	941	0.37 (0.03)	0.09 (0.01)	4410
440-700	-0.08 (0.16)	0.04 (0.01)	170	0.63 (0.20)	0.06 (0.01)	553	0.40 (0.05)	0.14 (0.01)	4569
700-950	0.56 (0.49)	0.11 (0.05)	31	-2.16 (0.99)	0.23 (0.06)	144	-0.00 (0.18)	0.24 (0.02)	1183
950-1200	-0.41 (1.03)	0.21 (0.08)	15	-1.67 (1.16)	0.21 (0.05)	65	0.31 (0.35)	0.26 (0.02)	553
1200-1900	-2.21 (1.28)	0.30 (0.18)	16	2.66 (0.99)	0.19 (0.08)	75	-0.15 (0.42)	0.29 (0.02)	638
1900-2600				3.40 (2.50)	0.11 (0.15)	17	0.70 (0.98)	0.38 (0.03)	214
2600-3200				-38.48 (0.00)	-0.02 (0.00)	4	2.41 (1.67)	0.41 (0.07)	70
3200-7500				8.24 (6.92)	-0.29 (0.18)	7	5.66 (3.38)	0.31 (0.07)	71

Table 5

Estimated β_E and β_{IR} for data binned by sign of equity excess return and OAS. One standard deviation error estimates are in parentheses. The data sample is as in Table 1. (Data for Figure 10.)

	Grouped by Starting OAS						Grouped by Ending OAS					
	Positive Equity Returns			Negative Equity Returns			Positive Equity Returns			Negative Equity Returns		
OAS Range	β_{IR}	β_E	N	β_{IR}	β_E	N	β_{IR}	β_E	N	β_{IR}	β_E	N
0-50 bp	1.03 (0.00)	0.00 (0.00)	11030	0.99 (0.01)	0.01 (0.00)	7748	1.03 (0.00)	0.01 (0.00)	11129	1.04 (0.00)	-0.01 (0.00)	7376
50-70	1.01 (0.00)	0.00 (0.00)	13429	0.96 (0.00)	0.02 (0.00)	9948	1.00 (0.00)	0.01 (0.00)	13557	1.01 (0.00)	0.00 (0.00)	9404
70-90	0.99 (0.00)	-0.00 (0.00)	12005	0.90 (0.01)	0.02 (0.00)	10072	0.97 (0.00)	0.01 (0.00)	11985	0.97 (0.01)	0.01 (0.00)	9886
90-110	0.96 (0.01)	-0.00 (0.00)	9474	0.87 (0.01)	0.02 (0.00)	8974	0.93 (0.01)	0.01 (0.00)	9444	0.92 (0.01)	0.01 (0.00)	8970
110-130	0.90 (0.01)	-0.00 (0.00)	8154	0.87 (0.01)	0.02 (0.00)	8222	0.91 (0.01)	0.01 (0.00)	8243	0.89 (0.01)	0.02 (0.00)	8102
130-170	0.85 (0.01)	0.00 (0.00)	12987	0.83 (0.01)	0.02 (0.00)	12677	0.85 (0.01)	0.01 (0.00)	12967	0.82 (0.01)	0.01 (0.00)	12732
170-210	0.81 (0.01)	0.01 (0.00)	8214	0.83 (0.01)	0.03 (0.00)	7762	0.82 (0.01)	0.01 (0.00)	8045	0.81 (0.01)	0.02 (0.00)	7992
210-250	0.77 (0.02)	0.02 (0.00)	4947	0.85 (0.02)	0.04 (0.00)	4492	0.78 (0.02)	0.00 (0.00)	4833	0.84 (0.02)	0.03 (0.00)	4697
250-350	0.66 (0.02)	0.02 (0.00)	5717	0.60 (0.03)	0.07 (0.00)	5872	0.62 (0.02)	0.02 (0.00)	5736	0.69 (0.02)	0.04 (0.00)	5754
350-440	0.43 (0.04)	0.03 (0.00)	2531	0.47 (0.05)	0.12 (0.01)	3074	0.44 (0.05)	0.04 (0.00)	2644	0.48 (0.05)	0.06 (0.00)	2978
440-700	0.38 (0.07)	0.08 (0.01)	2404	0.59 (0.07)	0.18 (0.01)	2888	0.24 (0.07)	0.10 (0.01)	2345	0.39 (0.05)	0.14 (0.01)	3252
700-950	0.66 (0.24)	0.13 (0.01)	547	0.45 (0.22)	0.33 (0.02)	811	0.27 (0.31)	0.16 (0.02)	496	-0.19 (0.14)	0.18 (0.01)	979
950-1200	0.19 (0.48)	0.19 (0.02)	260	0.82 (0.37)	0.33 (0.03)	373	1.11 (0.66)	0.18 (0.03)	249	0.24 (0.30)	0.26 (0.02)	424
1200-1900	1.46 (0.60)	0.22 (0.02)	299	0.93 (0.50)	0.40 (0.03)	430	0.84 (0.69)	0.25 (0.02)	241	0.51 (0.41)	0.37 (0.02)	562
1900-2600	2.49 (1.14)	0.29 (0.02)	90	2.82 (1.11)	0.55 (0.05)	143	2.31 (1.68)	0.30 (0.05)	69	-0.77 (0.60)	0.43 (0.03)	207
2600-3200	1.67 (2.43)	0.34 (0.08)	33	5.17 (2.54)	0.57 (0.07)	41	3.28 (3.13)	-0.03 (0.19)	22	0.28 (1.07)	0.59 (0.04)	76
3200-7500	-2.35 (8.43)	0.36 (0.11)	37	11.75 (3.36)	0.46 (0.08)	42	2.62 (13.85)	0.04 (0.18)	29	0.15 (2.01)	0.65 (0.05)	102

Table 6

Estimated common-factor and specific equity betas for data binned by sign of equity excess return and OAS. The columns marked $\beta_E - CF$ and $\beta_E - SP$ are the common factor and specific return betas. (Data for Figure 11.)

	Positive Equity Returns				Negative Equity Returns			
OAS Range	β_{IR}	β_E-CF	β_E-SP	*N*	β_{IR}	β_E-CF	β_E-SP	*N*
0-50 bp	1.03 (0.00)	0.00 (0.00)	-0.00 (0.00)	11030	0.98 (0.01)	0.02 (0.00)	0.00 (0.00)	7748
50-70	1.00 (0.00)	0.00 (0.00)	-0.00 (0.00)	13429	0.96 (0.00)	0.02 (0.00)	0.01 (0.00)	9948
70-90	0.99 (0.00)	0.00 (0.00)	-0.00 (0.00)	12005	0.90 (0.01)	0.03 (0.00)	0.01 (0.00)	10072
90-110	0.96 (0.01)	0.00 (0.00)	-0.00 (0.00)	9474	0.89 (0.01)	0.04 (0.00)	0.01 (0.00)	8974
110-130	0.90 (0.01)	-0.00 (0.00)	-0.00 (0.00)	8154	0.89 (0.01)	0.03 (0.00)	0.01 (0.00)	8222
130-170	0.85 (0.01)	0.00 (0.00)	0.01 (0.00)	12987	0.84 (0.01)	0.03 (0.00)	0.01 (0.00)	12677
170-210	0.82 (0.01)	0.02 (0.00)	0.01 (0.00)	8214	0.84 (0.01)	0.03 (0.00)	0.02 (0.00)	7762
210-250	0.77 (0.02)	0.03 (0.00)	0.01 (0.00)	4947	0.86 (0.02)	0.05 (0.01)	0.03 (0.00)	4492
250-350	0.67 (0.02)	0.03 (0.00)	0.01 (0.00)	5717	0.62 (0.03)	0.09 (0.01)	0.07 (0.01)	5872
350-440	0.45 (0.04)	0.05 (0.01)	0.02 (0.00)	2531	0.56 (0.05)	0.15 (0.01)	0.11 (0.01)	3074
440-700	0.42 (0.09)	0.10 (0.02)	0.07 (0.01)	2404	0.80 (0.07)	0.23 (0.01)	0.15 (0.01)	2888
700-950	1.00 (0.26)	0.25 (0.03)	0.08 (0.01)	547	0.78 (0.25)	0.39 (0.03)	0.31 (0.02)	811
950-1200	0.44 (0.48)	0.27 (0.04)	0.16 (0.03)	260	1.08 (0.43)	0.36 (0.04)	0.31 (0.03)	373
1200-1900	2.44 (0.69)	0.46 (0.06)	0.14 (0.02)	299	2.24 (0.68)	0.56 (0.05)	0.36 (0.03)	430
1900-2600	3.05 (1.34)	0.36 (0.09)	0.26 (0.03)	90	5.90 (1.51)	0.80 (0.08)	0.46 (0.05)	143
2600-3200	3.02 (3.91)	0.53 (0.31)	0.30 (0.10)	33	5.56 (3.03)	0.61 (0.13)	0.54 (0.10)	41
3200-7500	-2.36 (9.13)	0.36 (0.29)	0.36 (0.12)	37	9.14 (3.67)	0.32 (0.13)	0.50 (0.09)	42

Table 7

Estimated interest rate for data binned by sector and OAS. (Data for Figure 12.)

	Financial			Industrial			Utility		
OAS Range	β_{IR}	β_E	N	β_{IR}	β_E	N	β_{IR}	β_E	N
0-50 bp	1.05 (0.01)	-0.00 (0.00)	2676	1.00 (0.00)	0.00 (0.00)	9613	0.99 (0.00)	0.01 (0.00)	6489
50-70	1.01 (0.01)	-0.00 (0.00)	2733	0.98 (0.00)	0.01 (0.00)	11157	0.97 (0.00)	0.01 (0.00)	9487
70-90	0.99 (0.01)	-0.00 (0.00)	2281	0.94 (0.01)	0.01 (0.00)	11438	0.90 (0.01)	0.01 (0.00)	8358
90-110	0.95 (0.03)	0.00 (0.00)	1574	0.90 (0.01)	0.01 (0.00)	9682	0.90 (0.01)	0.01 (0.00)	7192
110-13	0.89 (0.03)	-0.01 (0.00)	1388	0.87 (0.01)	0.01 (0.00)	8826	0.86 (0.01)	0.01 (0.00)	6162
130-170	0.89 (0.02)	-0.01 (0.00)	2932	0.83 (0.01)	0.01 (0.00)	14443	0.81 (0.01)	0.01 (0.00)	8289
170-210	0.89 (0.03)	0.00 (0.01)	1851	0.80 (0.01)	0.02 (0.00)	9709	0.81 (0.01)	0.03 (0.00)	4416
210-250	0.96 (0.04)	0.02 (0.01)	1029	0.75 (0.01)	0.02 (0.00)	5889	0.81 (0.02)	0.03 (0.00)	2521
250-350	0.79 (0.06)	0.06 (0.01)	991	0.50 (0.02)	0.04 (0.00)	8220	0.71 (0.02)	0.04 (0.01)	2378
350-44	0.80 (0.10)	0.14 (0.02)	494	0.22 (0.03)	0.07 (0.00)	4585	0.81 (0.12)	0.11 (0.02)	526
440-700	0.64 (0.21)	0.17 (0.02)	342	0.29 (0.05)	0.13 (0.01)	4513	0.69 (0.19)	0.12 (0.02)	437
700-950	0.21 (0.48)	0.27 (0.04)	146	-0.11 (0.19)	0.23 (0.02)	1082	-0.43 (0.43)	0.23 (0.04)	130
950-1200	1.85 (0.71)	0.35 (0.06)	78	-0.07 (0.37)	0.25 (0.02)	502	-0.93 (0.74)	0.15 (0.05)	53
1200-1900	1.32 (0.84)	0.43 (0.05)	103	0.02 (0.46)	0.28 (0.02)	597	-1.30 (2.51)	0.19 (0.10)	29
1900-2600	4.19 (1.27)	0.61 (0.05)	38	0.50 (1.01)	0.34 (0.03)	187	-1.94 (11.53)	0.35 (0.13)	8
2600-3200				0.31 (1.86)	0.34 (0.05)	60			
3200-7500				3.79 (6.45)	0.26 (0.11)	42			

Table 8

Estimated interest rate and equity betas for bonds rated BB+/Ba1 and lower, grouped by whether or not they were below investment grade at issue. (Data for Figure 13.)

	High-Yield at Issue			Fallen Angel		
OAS Range	β_{IR}	β_E	N	β_{IR}	β_E	N
0-50 bp	0.97 (0.52)	-0.08 (0.04)	13			
50-70	0.74 (0.19)	0.01 (0.01)	30			
70-90	0.98 (0.15)	-0.02 (0.01)	89	0.82 (0.17)	-0.06 (0.04)	8
90-110	0.80 (0.14)	-0.01 (0.01)	139	0.93 (0.11)	0.01 (0.01)	28
110-130	0.88 (0.07)	-0.00 (0.01)	207	0.92 (0.14)	0.01 (0.01)	40
130-170	0.70 (0.06)	0.00 (0.00)	577	0.79 (0.08)	0.02 (0.01)	107
170-210	0.70 (0.07)	0.01 (0.01)	719	0.67 (0.12)	0.05 (0.03)	59
210-250	0.41 (0.08)	0.01 (0.00)	795	0.40 (0.24)	0.00 (0.01)	77
250-350	0.19 (0.04)	0.02 (0.00)	2734	0.74 (0.11)	0.02 (0.01)	434
350-440	-0.05 (0.06)	0.06 (0.01)	2080	0.26 (0.09)	0.03 (0.01)	371
440-700	-0.20 (0.07)	0.11 (0.01)	1789	0.50 (0.11)	0.12 (0.02)	569
700-950	0.11 (0.32)	0.21 (0.02)	509	0.14 (0.44)	0.26 (0.03)	170
950-1200	-0.82 (0.50)	0.20 (0.02)	257	1.18 (1.01)	0.30 (0.03)	74
1200-1900	-0.20 (0.58)	0.24 (0.02)	275	1.27 (1.65)	0.31 (0.06)	89
1900-2600	0.21 (1.79)	0.38 (0.05)	95	3.32 (4.04)	0.36 (0.10)	23
2600-3200	1.03 (2.28)	0.44 (0.11)	37	19.16 (26.43)	0.14 (0.41)	7
3200-7500	-20.00 (9.89)	0.18 (0.18)	21	9.65 (12.05)	0.28 (0.40)	7